Chapter 6

Trade

The previous two chapters have concerned techniques at a single location that are used to produce goods for local consumption. We now introduce the notion that different locations have different techniques and can exchange goods that they produce using these techniques.

Most of our analysis treats locations as Ricardian countries, of which there are $N$. Techniques for producing any good differ across countries while inputs are mobile across available techniques within, but not between, countries. While goods are in principle tradable across countries we allow for trade costs and, in some of the analysis, entry costs as well. A country $i$, $i = 1, ..., N$, is thus defined by the following set of parameters:

1. Input costs are simply reflected in the cost of an input bundle, which we denote
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\( w_i \). We turn to the determination of \( w_i \) through market clearing later in this chapter.

2. Trade costs reflect the added expense of delivering to and purchasing from other countries. In particular, we make the iceberg assumption that delivering a unit of a good from country \( i \) to country \( n \) requires the resources needed to produce \( d_{ni} \geq 1 \) units for local delivery (hence \( d_{ii} = 1 \)). We impose the triangle inequality, that for any third country \( h \), \( d_{ni} \leq d_{nh}d_{hi} \).\(^1\)

3. Entry costs reflect the cost of setting up sales to a particular market \( n \). To keep the model as stark as possible, we assume that all sources face the same entry requirement \( F_n \geq 0 \) in each market \( n \) (even the local producers), involving local inputs. Hence the entry cost \( E_n = w_n F_n \) is the same for everyone trying to sell in country \( n \). Note the distinction with the trade models of monopolistic competition discussed in Chapter 3, in which firms face a fixed cost of setting up production, but not of entering specific markets.

4. A country's state of technology \( T_i \) reflects the number of ideas that have arrived there. Each country has an independent arrival process, with the quality of each idea drawn independently from the Pareto distribution with parameter \( \theta \) (treated

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\(^1\) Arbitrage would eliminate violations of the triangle inequality since \( h \) would emerge as an entrepot, so that \( d_{ni} = d_{nh}d_{hi} \). Chapter 3 discusses the analytic convenience, as well as the limitations, of the iceberg specification of transport costs.
as common across countries). We turn to the evolution of $T_i$ over time in Chapter 7.

While we allow countries to differ in the number of ideas that they have had, we assume that the underlying distribution from which the qualities of ideas are drawn is the same everywhere. Specifically, the quality of each idea is drawn independently from a Pareto distribution with parameter $\theta$, where $\theta$ is the same around the world, as is the range of potential goods $j \in [0, J]$.

### 6.1 Cost Distributions in the Open Economy

We can introduce international trade into the analysis very seamlessly by reformulating Proposition 4.1 about the distribution of unit costs.

Consider techniques that provide country $n$ with some good $j$ at unit cost less than $c$. As we derived in Chapter 4, the number of local techniques that can do so is distributed Poisson with parameter $(T_n/J) w_n^{-\theta} c^\theta$. If $n$ can’t import, using these techniques is the only way for it to get good $j$.

Now consider some other country $i$ with its own techniques for making good $j$. By analogy, the number of these techniques that can produce good $j$ for local delivery at unit cost less than $c$ is distributed Poisson with parameter $(T_i/J) w_i^{-\theta} c^\theta$. Say that country $n$ can import good $j$ from country $i$. From the perspective of country $n$, country
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$i$'s input cost is not $w_i$, but rather $w_i d_n$, so that the number of $i$'s techniques for making good $j$ available to $n$ at unit cost less than $c$ is distributed Poisson with parameter $(T_i / J)(w_i d_n)^{-\theta} c^\theta$. Adding $i$'s techniques to the local ones, the total number available in $n$ is distributed Poisson with parameter $\left[ (T_n / J) w_n^{-\theta} + (T_i / J)(w_i d_n)^{-\theta} \right] c^\theta$, since the sum of independent Poisson draws is itself distributed Poisson. The probability $\pi_{ni}$ that one of these techniques is foreign is just $i$'s fraction of this parameter:

$$\pi_{ni} = \frac{(T_i / J)(w_i d_n)^{-\theta} c^\theta}{(T_n / J) w_n^{-\theta} + (T_i / J)(w_i d_n)^{-\theta}} = \frac{T_i (w_i d_n)^{-\theta}}{T_n w_n^{-\theta} + T_i (w_i d_n)^{-\theta}}.$$  

Note that the probability does not depend on $J$ or $c$. Hence the probability that a technique is foreign is the same regardless of the associated unit cost.

Extending this reasoning to a world of $N$ countries we define:

$$\Phi_n = \sum_{i=1}^{N} T_i (w_i d_n)^{-\theta}. \quad (6.1)$$

This expression summarizes what the history of the arrival of ideas around the world, along with input costs and trade costs, implies for the distribution of unit costs in any location $n$. We use to provide an open economy version of proposition 4.1.

**Proposition 8** Given $\Phi_n$: (i) The number of techniques providing unit cost less than $c$ for country $n$ is distributed Poisson with parameter $(\Phi_n / J) c^\theta$. (ii) The expected number of such techniques is $(\Phi_n / J) c^\theta$. (iii) The probability $\pi_{ni}$ that a technique providing unit cost less than $c$ is from country $i$ is:

$$\pi_{ni} = \frac{T_i (w_i d_n)^{-\theta}}{\Phi_n}. \quad (6.2)$$

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which is independent of \( c \).

(iv) The conditional distribution of unit costs provided by techniques from country \( i \) in country \( n \) is:

\[
\Pr[C \leq c'|C \leq c] = \Pr \left[ Q \geq \frac{w_id_{ni}}{c'} | Q \geq \frac{w_id_{ni}}{c} \right] = (c'/c)^\theta \quad c' \leq c. \tag{6.3}
\]

Note from (6.3) that the conditional distribution of costs depends only on the parameter \( \theta \), and not on any parameter specific to country \( i \) or \( n \). In particular, conditional on a technique delivering a unit cost to market \( n \) less than \( c \), the distribution of the unit cost does not depend on the source country \( i \).

Since all techniques available to a location, through local production or imports, provide the same conditional distribution of unit cost, what differs across locations is simply their number, as reflected in the term \( \Phi_n \), and their origin, as reflected in \( \pi_{ni} \). The term \( \Phi_n \) defined in (6.1) is the open economy version of (4.3) of Chapter 4. In the open economy \( \Phi_n \) reflects not only country \( n \)'s own state of technology \( T_n \), but the states around the world, tempered by input and trade costs. The more remote country \( n \) (as implied by higher \( d_{ni} \)'s) the lower its \( \Phi_n \).

Note that the conditional distribution (6.3) of unit cost is the same as (4.4) for the closed economy. Hence all of our results from Chapter 4 survive for each country \( n \), with each country having its own \( \Phi = \Phi_n/J \) governing the joint distribution of the ordered costs \( C_n^{(1)} \leq C_n^{(2)} \leq C_n^{(3)} \leq \ldots \) of each good \( j \) there.

Since the probability \( \pi_{ni} \) that a particular country \( i \) is the source of a version at cost less than \( c \) is the same for all \( c \), it is the same across all rankings of costs \( k \). So
\( \pi_{ni} \) is the probability that country \( i \) can deliver some good \( j \) at the lowest unit cost, second lowest cost, etc. Under a very general assumption about market structure, \( \pi_{ni} \) will also be both: (i) the likelihood that a version of good \( j \) bought by country \( n \) comes from country \( i \) and (ii) the expected share of country \( n \)'s expenditure on good \( j \) bought from country \( i \).

Consider any preference and market structure with the property that the price (or price index) for good \( j \) in country \( n \), \( p_n(j) \), depends only on the realization of the ordered unit costs for good \( j \), \( C_n^{(1)}(j) \leq C_n^{(2)}(j) \leq C_n^{(3)}(j) \leq \ldots \) and, possibly, aggregate magnitudes for market \( n \) such as the price index and total expenditure. That is, given costs and aggregate magnitudes, other supplier characteristics, such as nationality, are inconsequential for prices. We refer to this property as “anonymity.”

> From (6.3), the conditional distribution of unit costs doesn’t depend on the nationality of the source. Combining this result with any “anonymous” market structure yields the result that the probability \( \pi_{ni} \) that a unit cost is represented by a technique from country \( i \) is also the likelihood that \( n \) buys a version of good \( j \) from country \( i \) and the expected share of its spending on good \( j \) going to country \( i \). In the case of perfect or Bertrand competition only the low cost supplier is active, so that \( \pi_{ni} \)

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\(^2\)All of the market structures considered in the previous chapter are “anonymous” in this sense. In the cases of perfect or monopolistic competition price is proportional to \( C^{(1)} \) while with Bertrand competition it depends on \( C^{(1)} \) or \( C^{(2)} \). With Cournot competition higher-order costs can matter as well. But given the \( C^{(k)} \)'s, source doesn’t matter.
is the probability that a producer from country $i$ is the supplier of good $j$ to country $n$. In the case of monopolistic competition, it is the probability that any variety comes from country $i$, since all sources face the same cutoff cost in market $n$, $\tau_n$.\footnote{To illustrate the role of anonymity, consider two sources of violation: (i) preferences with the Armington property that source enters directly into utility and (ii) market structures in which sellers from a particular country (say home) are Stackelberg leaders in setting quantity or price.}

## 6.2 Aggregate Implications

Having characterized the implications of the model for a particular good $j$ we now integrate across goods to explore the aggregate implications of our model. As before, we treat $w_i$ as pertaining to all goods $j$ that might be produced in source country $i$. In addition we treat the trade cost parameters $d_{ni}$ as common across any good shipped from $i$ to $n$. An immediate implication is that $\Phi_n$ defined in (6.1), as well as the $\pi_{ni}$ defined in (6.2), are also common across all goods. As before, the probability distribution of the efficiency for any particular good $j$ is also the distribution of efficiency draws across goods.

Our results for the closed economy in the previous chapter apply, with $\Phi_n$ redefined as (6.1). In particular, the price index $P_n$ in country $n$ remains:

$$P_n = \Gamma_n \Phi_n^{-1/\theta}$$  \hspace{1cm} (6.4)

where $\Gamma_n$ can be derived explicitly for the various market structures considered in the
previous chapter. In the open economy $\Phi_n$, reflects not only the country’s own state of technology $T_n$, but the states around the world, tempered by input and trade costs. The more remote country $n$ is (as implied by higher $d_n$'s) the lower its $\Phi_n$ and, hence, the higher its $P_n$.

As derived in Chapter 5, with perfect and Bertrand competition, since the range of goods is fixed, $\Gamma_n$ is the same across countries and depend only on the parameters $\sigma$ and $\theta$ and the range of goods $J$.

While the parameter $\Phi_n$ summarizes all that the parameters of the model imply for price differences across countries, the $\pi_{ni}$ indicate the direction of trade. In particular, since $\pi_{ni}$ is the probability that a purchase by country $n$ is from $i$, $\pi_{ni}$ becomes the fraction of purchases that $n$ makes from $i$. Since $\pi_{ni}$ is country $i$'s expected share in country $n$'s spending on any particular good, it is the fraction of $n$'s total spending that is spent on goods from $i$.

The result that $\pi_{ni}$ is the fraction of goods bought from $i$ follows immediately from Part (iii) of Proposition 6.1, since it is the probability that any single purchase from $i$. We can thus divide the measure of goods supplied in country $n$ into the range supplied by each source country $i$. By Part (iv) of the Proposition, conditional on a country supplying a particular good, its cost is drawn from the same distribution as a supplier from any other source. Moreover, under anonymity, conditional on the realization of its cost, the distribution of its price is the same. Since the price distribution doesn’t
depend on source, the fraction of spending going to \( i \) is the same as the fraction of goods bought from \( i \).

This result provides a link between \( \pi_{ni} \) and trade shares, that:

\[
\pi_{ni} = \frac{X_{ni}}{X_n}
\]

where \( X_n \) is total spending by \( n \) and \( X_{ni} \) is the value of imports from \( i \) (including domestic production when \( i = n \)). We exploit this simple and direct link between the theory and data in several of our applications below.

Filling in the determinants of \( \pi_{ni} \) gives us an expression for bilateral trade shares:

\[
\frac{X_{ni}}{X_n} = \frac{T_i(w_id_{ni})^{-\theta}}{\sum_{h=1}^{N} T_h(w_hd_{nh})^{-\theta}}.
\] (6.5)

This expression for trade shares resembles those for Armington (3.3) and for monopolistic competition (3.13). There are two important differences. First, the scale measure for country \( i \) is no longer its share in preferences or its labor force, but its state of technology \( T_i \), reflecting the history of ideas that have arrived in the country. Second, the elasticity parameter is no longer the elasticity of substitution in preferences but the parameter \( \theta \) of the Pareto distribution for the quality of ideas, reflecting their heterogeneity. A greater value of \( \theta \) means that ideas are more similar, so that comparative unit costs differ less from good to good. Hence, with a greater \( \theta \), a given increase in trade costs \( d_{ni} \) will cause country \( n \) to switch its sourcing of more goods away from country \( i \). Unlike Armington or monopolistic competition, adjustment is not at the
extensive margin, how much of each good is purchased, but at the intensive margin, the range of goods purchased.

6.3 Gravity

Having drawn the analogy with Armington and monopolistic competition, we can put expression (6.5) through similar paces to obtain various gravity-like expressions. First, we can write total sales $Y_i$ of country $i$ as:

$$Y_i = \sum_{n=1}^{N} X_{ni} = T_i w_i^{-\theta} \sum_{n=1}^{N} \frac{d_{ni}^{-\theta} X_n}{\Phi_n} = T_i w_i^{-\theta} \Xi_i$$

(6.6)

where:

$$\Xi_i = \sum_{m=1}^{N} \frac{d_{mi}^{-\theta} X_m}{\Phi_m}$$

(6.7)

reflects country $i$'s market potential, similar to the expressions $\Xi_i$ derived for Armington and monopolistic competition in Chapter 3.

Solving (6.6) for $T_i w_i^{-\theta}$ and substituting this expression and the definition of $\Phi_n$ (6.1) into (6.5) gives:

$$X_{ni} = \frac{Y_i X_n d_{ni}^{-\theta}}{\Xi_i \Phi_n}$$

(6.8)

an expression much like the gravity equation derived from Armington (3.6) and for monopolistic competition (3.14). The difference is that the term $\Phi_n$ enters in place of $P_n^{-\theta}$ both indirectly through $\Xi_i$ and directly. In perfect and Bertrand competition
these terms are interchangeable (since $\gamma^{PC}$ and $\gamma^{MC}$ cancel) so we have yet again the identical equation.

With monopolistic competition, there is a substantive difference, however, since the price level $P_n$ depends not only on technology and input costs but on market size relative to entry cost. Substituting the price index for monopolistic competition (5.9) into (6.8) and $\Xi_i$ gives us:

$$X_{ni} = \frac{Y_i X_n}{\Xi_i} \left( \frac{d_{mi}}{P_n} \right)^{-\theta} \left( \frac{X_n}{\sigma E_n} \right)^{[\theta-(\sigma-1)]/(\sigma-1)}.$$

Given its price level, a large country, imports more than in proportion to its size. Low prices due to variety, rather than due to low cost competitors, are not a deterrent to sales there.

### 6.4 The Gains from Trade

In this section and the one that follows we will take labor to be the only input. Hence $w_i$ is the wage in country $i$. In the last section of this chapter we generalize the analysis to allow for intermediates.

The model provides an immediate expression for the gains from trade, in the form of higher real wages, as a function of trade shares. Using the price index (6.4) we can rewrite equation (6.2) for $n = i$ as:

$$\frac{w_i}{P_i} = \frac{1}{\Gamma_i} \left( \frac{T_i}{\pi_{ii}} \right)^{1/\theta}$$
where $\pi_{ii}$ is the fraction of spending that $i$ does at home. Under autarky, $\pi_{ii} = 1$ and we have our expression for the real wage in the closed economy as in the previous chapter. Trade augments a country's effective technology by a factor of $1/\pi_{ii}$. Country's that trade more, gain more. Taking a value of $\theta = 8$ (close to one of our estimates below), a country that has an import share of 0.2 would suffer a 2.8 percent decline in its real wage from a move to autarky. The reasoning here is analogous to price indices constructed to account for the introduction of new goods over time. Such price indices adjust the price index for goods available in all periods by the fraction of goods each period that are available in all periods (see Feenstra, 1994).

With perfect and Bertrand competition $\Gamma_i$ is just a constant. Local technology and openness are the only determinants of cross-country differences in real wages. With monopolistic competition and entry costs, we get:

$$\frac{w_i}{P_i} = \frac{1}{\gamma^{MC}} \left( \frac{X_i}{\sigma E_i} \right)^{[\theta-(\sigma-1)]/[\theta(\sigma-1)]} \left( \frac{T_i}{\pi_{ii}} \right)^{1/\theta}$$

An additional factor is market size relative to the entry cost. A larger market can sustain greater variety, raising welfare. To give some sense of magnitudes, combine our value of $\theta = 8$ with an elasticity of substitution $\sigma = 5$. The elasticity of the price level with respect to $X_i/E_i$ is then $1/8$. Note that technology, trade, and market size affect the real wage multiplicatively, allowing for a clean decomposition of their effects.

This analysis takes $\pi_{ii}$, $X_i$ and $E_i$ as given, so has not dug down to fundamentals. To perform this task we turn to markets for inputs into production.
6.5 Labor-Market Equilibrium

Simplest is to make the standard Ricardian assumption that labor is the only input. Consider the condition for labor-market equilibrium in each country, choosing one country's wage as numeraire. We provide a stripped-down analysis here. Alvarez and Lucas (2004) tackle a more general set up and also provide conditions for uniqueness of the equilibrium wage vector.

Let $L_i$ denote the number of workers available for production (or, with entry costs, for overhead as well) in country $i$. Total spending on labor in country $i$ is:

$$w_i L_i = (1 - \delta) \sum_{n=1}^{N} \frac{(w_i d_{ni})^{-\theta} T_i}{\Phi_n} X_n \quad i = 1, ..., N$$

where $\delta$ is the profit share. In the case of perfect competition $\delta = 1$, while the previous chapter derived expressions for $\delta$ in the cases of Bertrand and monopolistic competition.

With balanced trade, spending $X$ is equal to labor income plus profit, so that:

$$X_n = \frac{1}{1 - \delta} w_n L_n.$$

Hence we can write our labor-market equilibrium conditions as:

$$w_i L_i = w_i^{-\theta} T_i \sum_{n=1}^{N} \frac{d_{ni}^{-\theta} w_n L_n}{\sum_{k=1}^{N} (w_k d_{nk})^{-\theta} T_k} \quad i = 1, ..., N$$

(Magically, the profit share has disappeared.) In equilibrium the wages $w$ satisfy this set of equations. In general there is no closed-form solution, but a numerical solution is easy to obtain even for a realistically large $N$. Note that the conditions for labor market equilibrium are the same across market structures.
Note that we can use our definition of market potential to reformulate this expression as:

\[ w_i = \frac{\Xi_i}{L_i}, \quad i = 1, \ldots, N \]

the wage is equal to market potential divided by the labor force. Since market potential depends on wages everywhere, this expression does not constitute a closed-form solution.

A special case provides insight into what relative wages depend on. Consider the case of "frictionless" trade in which \( d_{ni} = 1 \) for all \( i \) and \( n \). The summation term in expression (6.9) is then the same for all countries \( i \). Taking the ratio of the wages in two countries \( i \) and \( k \) gives:

\[ \frac{w_i}{w_k} = \left( \frac{T_i/L_i}{T_k/L_k} \right)^{1/(1+\theta)}. \]

With all \( d_{ni} = 1 \) price levels are the same everywhere without entry costs. Hence this ratio is also the ratio of real wages for the cases of perfect and Bertrand competition. (In the case of monopolistic competition the price levels will still differ across markets of different size, since larger markets attract more sellers.)

Note that without trade costs relative wages depend on the state of technology relative to the labor force, with an elasticity \( 1/(1+\theta) \). In comparison, from the previous chapter, the ratio in the case of a closed world \( (d_{ni} \to \infty, \ n \neq i) \) is:

\[ \frac{w_i}{w_k} = \left( \frac{T_i}{T_k} \right)^{1/\theta}. \]
Since trade allows workers to specialize in a narrow range of goods, $T/L$ rather than $T$ is what matters for the relative wage. Moreover, in the open economy the benefit of an increase in a particular country’s $T$ is shared by others through lower prices, so that the elasticity of the relative wage with respect to the relative $T$ is lower.

### 6.6 Intermediates

The economic geography literature has emphasized the role of location not only for market potential, but also for production costs. We can do so in our framework by incorporating intermediate goods into the analysis. Assume that inputs combine labor and intermediate goods, with labor having a share $\beta$, and that intermediates are representative of goods generally, and that the same CES aggregator applies. The cost $w_i$ of a bundle of inputs in country $i$ is then proportional to $v_i^\beta P_i^{1-\beta}$ where now $v_i$ is the wage in country $i$. Using the expression for the price index, a condition relating prices around the world, given wages $v$, is then:

$$P_n^{-\theta} = \varepsilon \sum_{i=1}^{N} T_i \left( v_i^\beta P_i^{1-\beta} d_{ni} \right)^{-\theta} \quad n = 1, ..., N$$

(6.10)

where $\varepsilon = \beta^{-\beta} (1-\beta)^{-(1-\beta)}$. This expression shows how higher prices in one country spill-over to others through input costs.
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The condition for labor-market equilibrium becomes:

\[ v_i L_i^P = \left( v_i^\beta P_i^{1-\beta} \right)^{-\theta} T_i \sum_{n=1}^N \frac{d_{ni} \theta w_n L_n^P}{\sum_{k=1}^N \left( v_k^\beta P_k^{1-\beta} d_{nk} \right)^{-\theta} T_k} \quad i = 1, \ldots, N \]  \hspace{1cm} (6.11)

An equilibrium is a set of price indices \( P_i \) and wages \( v_i \) that solve (6.10) and (6.11). Again, while there is no closed-form solution for the general case, a numerical one is easy to obtain for a realistic number of countries.

This formulation delivers the result that more remote locations suffer not only from lack of access to foreign markets, but from higher input prices.
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References

