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Chapter 5

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Preferences and Market Structure

The previous chapter derived the distribution of unit costs implied by particular assumptions about technology, given the cost of inputs. To characterize the general equilibrium of the economy we need to make further assumptions about demand, market structure, and access to ideas.

We pursue a several alternatives. Before turning to them one by one, however, we state one result which holds very generally.

A feature of general equilibrium is the linear homogeneity of each price in all costs. That is, the unit of account is irrelevant for any real magnitude. Since any price index is a linear homogenous function of the prices of individual goods, the price index is itself linear homogenous in costs. It follows from Lemma 7 of the previous chapter

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that we can express any price index P_i as:

$$P_i = \Gamma_i \Phi_i^{-1/\theta} = \Gamma_i T_i^{-1/\theta} w_i \quad (5.1)$$

where Γ_i does not depend on either the state of technology T_i or the cost of inputs w_i . An immediate implication is that, if labor is the only input, the real wage is proportional to $T_i^{1/\theta}$, given Γ_i . In what follows we could assign the input bundle the role of numeraire, and set $w_i = 1$. We often find it clearer to make its role explicit, however. What is in Γ_i will depend on our specific assumptions about preferences and market structure, to which we now turn.

We limit ourselves to ones which (1) deliver relatively simple closed-form solutions and (2) relate to existing analysis and our applications. Specifically, we show how the cost structure derived in the previous chapter is consistent with various sets of assumptions that are common in the literature: perfect competition, Bertrand competition, monopolistic competition, and Cournot competition. We provide complete expressions for the Γ_i component of the price index for the first three. We also investigate the role of entry costs, which play a particularly important role in monopolistic competition.

In this chapter we consider the economy at only one moment, so we suppress the time indicator t from the previous chapter. Even though we consider only a closed economy, in anticipation of what follows on occasion we use the i subscript to distinguish magnitudes that can vary across location from universal ones. Chapter 6 shows how the

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analysis here extends very seamlessly to accommodate trade between different locations at a given moment. Chapter 7 examines ways of modeling how technologies evolve over time. While the analysis here is static, the various situations we consider can be thought of as reflecting different assumptions about the evolution and ownership of ideas.

The previous chapter derived features of the joint distribution of the unit costs $C^{(k)}(j)$ of a good j , where $k = 1, 2, \dots$ refers to the k 'th lowest unit cost. We continue to treat the set of available goods as an exogenous continuum $[0, J]$ indexed by j , although in the first three cases J plays no role, so we set $J = 1$.

We introduce the assumption that, for buyers, the elasticity of substitution between goods is σ . We also allow for the possibility that buyers may regard versions of good j produced at different unit costs as imperfect substitutes, with the elasticity of substitution among different versions of the same good $\sigma' \geq \sigma$ (that is, different versions of the same good are closer substitutes than different goods). Our assumptions about demand follow from preferences of the form:

$$U = \left[\int_0^J u(j)^{\sigma/(\sigma-1)} dj \right]^{(\sigma-1)/\sigma}$$

where:

$$u(j) = \left[\sum_{k=1}^{\infty} y^k(j)^{\sigma'/(\sigma'-1)} \right]^{(\sigma'-1)/\sigma'}$$

where $y^k(j)$ is consumption of the k 'th lowest-cost version of the j 'th good.¹

¹A straightforward extension, which we delay until Chapter 6, recognizes demand for intermediates as well as final goods.

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A consumer faces prices $p^k(j)$, $k = 1, 2, 3, \dots$; $j \in [0, J]$. If a version is not on the market we treat its price as infinite. With a total expenditure of X , the amount spent on version k of good j is:

$$X^k(j) = X \left(\frac{p^k(j)}{p(j)} \right)^{1-\sigma'} \left(\frac{p(j)}{P} \right)^{1-\sigma} . \quad (5.2)$$

where:

$$P = \left[\int_0^J p(j)^{1-\sigma} dj \right]^{1/(1-\sigma)} . \quad (5.3)$$

and:

$$p(j) = \left[\sum_{k=1}^{\infty} p^k(j)^{1-\sigma'} \right]^{1/(1-\sigma')} .$$

The resulting utility is X/P .

A problem could emerge if more than a finite number of versions of good j are available at finite prices. The expression for $p(j)$ could go to zero for $\sigma' < \infty$: The consumer's utility from good j explodes toward infinity due to the plethora of versions. If this phenomenon occurs for a finite measure of goods then P also goes to zero and overall utility explodes to infinity. We take two routes, either of which is sufficient, to rule this problem out. One restricts preferences, the other technology.

Setting $\sigma' \rightarrow \infty$ keeps $p(j)$ bounded even if an infinite number of $p^k(j)$ are finite. In this case consumers regard all versions of a product as perfect substitutes. Given prices, the introduction of a new version does not add to utility.

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To allow for finite σ' requires bounding the amount of variety that an economy can provide. Entry costs do the trick. We assume that serving a market requires employing a fixed quantity F of inputs. Only a finite number of versions of each good will be offered in equilibrium (since otherwise total production costs would be infinite). Whether or not there is an entry cost, a producer incurs a constant unit cost of production (the properties of which were derived in the previous chapter).

Having described technology and preferences, our two routes to bounding the problem guide how we proceed in introducing market structure.

With $\sigma' \rightarrow \infty$ producers of a good engage in head-to-head competition, with consumers buying only the version available at the lowest price. With the set of goods given by the measure $[0, J]$ there is no scope for a producer to carve out a new product. This case, without entry costs, connects with the Ricardian and quality ladders models discussed in Chapter 3. Hence, under this assumption, we consider perfect competition (which, in our constant unit cost world, is incompatible with an entry cost anyway) and Bertrand competition. The first emerges if ideas are freely available, the second if they are proprietary.

With an entry requirement $F > 0$, a particularly simple case emerges by setting $\sigma' = \sigma$. In this case every idea represents an equally distinct product in terms of demand. Consumers regard different versions of the same good as distinct as different goods. With proprietary ideas, this world connects with the literature on monopolistic

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competition.

Specifically, we analyze four different cases in turn:

1. Assuming $\sigma' \rightarrow \infty$, $F = 0$, and perfect competition delivers the Ricardian competitive model with a continuum of goods analyzed in Eaton and Kortum (2002) and Alvarez and Lucas (2004).
2. Assuming $\sigma' \rightarrow \infty$, $F = 0$, and Bertrand competition among proprietary owners of each idea delivers quality ladders. Kortum (1997), Eaton and Kortum (1999), and BEJK (2003) consider this case.
3. Assuming $\sigma' = \sigma$, $F > 0$, and monopolistic competition delivers a variant of the models developed in Helpman, Melitz, and Yeaple (2004), Chaney (2005), and Helpman, Melitz, and Rubinstein (2005).
4. Assuming $\sigma' \geq \sigma$ and $F \geq 0$ (with either $F > 0$ or $\sigma' \rightarrow \infty$) and Cournot competition delivers a model related to Atkeson and Burstein (2005).

We characterize the full equilibrium for the first three cases, and provide some results on the fourth. There are many other possibilities, of course, that might prove useful in other applications.

We speak of inputs as labor, with endowment L_i and wage w_i . We think of w_i as numeraire but often find it more transparent to include it explicitly in our expressions, especially in anticipation of our extension to international trade.

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Before proceeding we place two restrictions on σ . In the case of monopolistic competition, we assume $\sigma > 1$ to assure that a monopolist's markup is bounded. In all four cases we also need to restrict how heterogeneous goods are in terms of cost (as reflected by the parameter θ from the previous chapter) relative to how heterogeneous they are in preferences (as reflected by the elasticity of substitution σ). Specifically we will need $\sigma < \theta + 1$ in order to apply (4.10) and (4.11) from the previous chapter. A low value of θ implies that cost (and, hence, in our applications, price) varies a great deal across goods j , while a high value of σ implies that goods are perceived as very similar. As $\theta \rightarrow \sigma - 1$ purchases become ever more concentrated in goods with low price, to the point at which, in the limit, the price index becomes undefined. As you will see, in each case we examine below, the price index is defined only when $\sigma < \theta + 1$.

A 5.1 Perfect Competition

As $\sigma' \rightarrow \infty$, consumers regard all varieties of each good j as equivalent. Under perfect competition only the lowest unit cost version of the good will be purchased, with price $p(j)$ equal to the lowest unit cost $C^{(1)}(j)$. The distribution of prices will thus correspond to the distribution of lowest costs given in (4.9). We now demonstrate:

Proposition 2 *Under perfect competition, given $\theta > \sigma - 1$, the CES price index is:*

$$P_i = \gamma^{PC} \Phi_i^{-1/\theta} \tag{5.4}$$

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where

$$\gamma^{PC} = \left[J^{1+(1-\sigma)/\theta} \Gamma \left(\frac{\theta - (\sigma - 1)}{\theta} \right) \right]^{1/(1-\sigma)}$$

and $\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy$ is the Gamma function.

The Chapter Appendix provides the proof of this proposition and of propositions 3 through 6.

Note that the term Γ_i in the general price index (5.1) reduces to γ^{PC} , which involves the only parameters, θ and σ and the range of goods J . It does not depend on location i or date t .

Recalling that $\Phi_i = T_i w_i^{-\theta}$, two observations are worth making: First, not surprisingly, the price level is homogeneous in the wage. Second, given the wage w_i , the parameter of the distribution of technology T_i affects prices with an elasticity $-1/\theta$. Hence the framework delivers a simple connection between the state of technology T_i and the real wage w_i/P_i :

$$\frac{w_i}{P_i} = \frac{1}{\gamma^{PC}} T_i^{1/\theta} \tag{5.5}$$

As we soon see, changes in assumptions about market structure affect the price index and the wage only through the parameter γ . Technology has the same role throughout.

Expenditure on good j with price $p(j)$ is:

$$X(j) = \left(\frac{p(j)}{P_i} \right)^{1-\sigma} X_i,$$

where, since there are no profits, $X_i = w_i L_i$.

We conclude with one further result for perfect competition that anticipates results that follow:

Remark 1 *With $\sigma > 1$, revenues $X(j)$ are greater for a good with a lower realization of cost $C^{(1)}(j)$.*

The result follows immediately from substituting $C^{(1)}(j)$ for $p(j)$ in (5.2), setting $\sigma' \rightarrow \infty$. Hence goods with lower costs occupy a larger share of expenditure.

A 5.2 Bertrand Competition

With $\sigma' \rightarrow \infty$, Bertrand competition, like perfect competition, means that only the lowest unit cost variety is sold. With each idea owned by a single producer, this supplier will charge a price equal to the lesser of the monopoly (Dixit-Stiglitz) markup over cost, and the unit cost of the second lowest cost supplier $C^{(2)}(j)$:

$$P(j) = \min \{ \bar{m}C^{(1)}(j), C^{(2)}(j) \}$$

where we refer to

$$\bar{m} = \frac{\sigma}{\sigma - 1}$$

as the Dixit-Stiglitz markup (the markup that would be charged by the owner of an idea for good j facing no competition from other varieties). Any potential variety of a good with unit cost ranked third or more is irrelevant to the market equilibrium. The

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implied markup, then, is:

$$M(j) = \frac{P(j)}{C^{(1)}(j)} = \min \left\{ \frac{C^{(2)}(j)}{C^{(1)}(j)}, \bar{m} \right\} \quad (5.6)$$

Unlike the case of monopolistic competition, the markup varies depending on the unit costs of the first and second lowest cost supplier, and will vary stochastically across goods. Applying (4.14) to (5.6) delivers:

Proposition 3 *Under Bertrand competition the distribution of the markup M is:*

$$\Pr [M \leq m] = F_M(m) = 1 - m^{-\theta}$$

for $m \leq \bar{m}$. With probability $\bar{m}^{-\theta}$ the markup is \bar{m} . The markup is independent of $C^{(2)}$

We now establish the following result on the price index and on the profit share of the economy:

Proposition 4 *Under Bertrand competition the price index is:*

$$P_i = \gamma^{BC} \Phi_i^{-1/\theta} \quad (5.7)$$

where:

$$\gamma^{BC} = \left[J^{1+(1-\sigma)/\theta} \left(1 + \frac{(\sigma-1)\bar{m}^{-\theta}}{\theta - (\sigma-1)} \Gamma \left(\frac{2\theta - (\sigma-1)}{\theta} \right) \right) \right]^{1/(1-\sigma)}.$$

As with perfect competition, the term Γ_i in the general price index reduces to a constant γ^{PC} depending only on the parameters, θ and σ , and the range J of goods.

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Under perfect competition there were, of course, no profits. That is not the case under Bertrand competition in which the owner of the lowest cost idea earns a rent equal to:

$$\pi(j) = (P(j) - C^{(1)}(j))(X(j)/P(j)) = (1 - M(j)^{-1})X(j). \quad (5.8)$$

For any individual producer this rent depends not only on the realization of her own cost, but that of the second lowest cost producer as well. However, averaging across all active producers, the profit share in the economy turns out to have a simple form. We now establish:

Proposition 5 *Under Bertrand competition aggregate profit is:*

$$\Pi_i = \delta^{BC} X_i$$

where

$$\delta^{BC} = \frac{1}{1 + \theta}$$

and

$$X_i = \frac{1 + \theta}{\theta} w_i L_i$$

is total spending.

It might come as a surprise that even though the markup is capped at $\bar{m} = \sigma/(\sigma - 1)$, the share of profit in the economy is independent of σ . The explanation is that while a higher value of σ limits the markup that any producer will charge, it also

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implies greater sales and hence higher profit to low cost sellers who are more likely to be constrained by \bar{m} , with the two effects cancelling out.

This result on the profit share of the economy is very useful in Chapter 7, where we endogenize the production of ideas in the quality ladders model. It implies a simple expression for the expected discounted value of an idea, and hence the return to innovative activity.

We conclude with a result on the relationship between cost, price, sales, and the markup.

Remark 2 *A lower unit cost $C^{(1)}(j)$ is associated with: (i) a lower price, (ii) with $\sigma > 1$ larger sales, and (iii) a higher markup.*

The first result follows from the fact that conditioning on a lower $C^{(1)}$ in the distribution (4.13) yields a distribution of $C^{(2)}$ that is worse (in the sense of stochastic dominance). Hence firms with a lower unit cost are more likely to face stronger competition so are likely to charge a lower price. The second result follows from a lower price leading to higher sales if $\sigma > 1$. The third result follows from (4.14): A lower $C^{(1)}$ implies a distribution of M that is better (in the sense of stochastic dominance).

In contrast with perfect competition and monopolistic competition (taken up next), a producer with a lower unit cost will, on average, charge a price that is not proportionately lower, thus charging a higher markup. The result implies a correlation between size and markups. A seller with lower cost is more likely both to sell more and

to earn a higher profit per sale.

A 5.3 Monopolistic Competition

For $\sigma' = \sigma$, buyers regard different varieties of a product as differentiated to the same extent as different products. As in the case of Bertrand competition, each producer has a monopoly on her idea. The markup over unit cost is now simply $\bar{m} = \sigma/(\sigma - 1)$. We introduce an entry requirement of F_i input bundles to serve the market. With an input cost w_i the entry cost is:

$$E_i = w_i F_i.$$

The number of active sellers is now endogenous. In what follows, we treat labor as the only input, distinguishing between production labor and overhead labor.

>From (4.8), the distribution of costs is the same as if each good were produced with a level of efficiency drawn from a Pareto distribution, as in the model of monopolistic competition of Helpman, Melitz, and Yeaple (2004), Chaney (2005), and Helpman, Melitz, and Rubinstein (2005). Hence this section provides a bridge between their work and ours.²

²In Eaton, Kortum, and Kramarz (2005) we take another path from our general framework to monopolistic competition. There we set $\sigma' \rightarrow \infty$, so that only the lowest-cost version is ever produced. From (4.9) the measure of goods with unit cost less than $C^{(1)} \leq c_1$ is:

$$JF_1(c_1) = J \left[1 - \exp \left(-\frac{\Phi}{J} c_1^\theta \right) \right].$$

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We establish the following result on the price index and about market entry:

Proposition 6 *Under monopolistic competition the price index in a market with total sales X_i is:*

$$P_i = \gamma^{MC} \left(\frac{X_i}{\sigma E_i} \right)^{-[\theta - (\sigma - 1)]/[(\sigma - 1)\theta]} \Phi_i^{-1/\theta} \quad (5.9)$$

where

$$\gamma^{MC} = \bar{m} \left[\frac{\theta}{\theta - (\sigma - 1)} \right]^{-1/\theta}.$$

Entry is profitable only for producers with cost $c \leq \bar{c}_i$ given by:

$$\bar{c}_i = \left(\frac{X_i}{\sigma E_i} \right)^{1/(\sigma - 1)} \frac{P_i}{\bar{m}}. \quad (5.10)$$

and the measure of active sellers H_i is:

$$H_i = \frac{X_i}{\sigma E_i} \frac{\theta - (\sigma - 1)}{\theta}. \quad (5.11)$$

In contrast to perfect and Bertrand competition with no entry cost, with monopolistic competition a larger market attracts a greater variety of sellers in proportion to X_i/E_i . A larger market thus provides more variety. The term Γ_i in the general price index (5.1) captures this effect. Not only does it depend on the term γ^{MC} , a function of σ , θ , and J , it also depends on the the variable profits that the market generates (X_i/σ) relative to the entry cost E_i with an elasticity:

$$-\frac{\theta - (\sigma - 1)}{\theta(\sigma - 1)} < 0$$

Taking the limit as $J \rightarrow \infty$ delivers $H(c)$ as in equation (4.8).

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Several additional points are worth noting:

1. Even though entry is endogenous, the number of sellers $H_i = H_i(\bar{c}_i)$ is independent of $\Phi_i = T_i w_i^{-\theta}$, the state of variable of costs as determined by the state of technology, the same as the case with perfect and Bertrand competition. An increase in Φ_i lowers the cutoff value \bar{c}_i , driving higher-cost sellers from the market.
2. The price level relates to the level of technology as in the case of perfect and Bertrand competition, falling with respect to the measure of ideas T_i with elasticity $1/\theta$.
3. The real wage w_i/P_i rises with these magnitudes with the same elasticity. Hence larger markets, by accommodating more entrants and hence delivering more variety, provide a higher real wage, a standard result in monopolistic competition. A higher T_i delivers a higher real wage through lower-cost goods, not through more goods.

The price index properly takes into account the range of goods sold as well as what they cost. If we were to condition on the range of goods sold, the price index P_i^S would be:

$$\begin{aligned}
 P_i^S &= \left[\frac{1}{H_i} \int_0^{\bar{c}_i} (\bar{m}c)^{1-\sigma} dH_i(c) \right]^{1/(1-\sigma)} \\
 &= \bar{m} \left[\frac{\theta}{\theta - (\sigma - 1)} \right]^{1/(1-\sigma)} \bar{c}_i.
 \end{aligned}$$

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A market that is larger relative to the entry cost of entry (having a larger X_i/E_i) attracts more entrants, but the marginal entrants have higher costs. Hence the average price of a good sold is higher, even though the true price index is lower.³

As in the previous cases, a lower-cost firm will sell more, since its price is lower. As with perfect competition, but unlike Bertrand competition, a lower unit cost is fully reflected in a lower price, since the markup is constant.

We conclude with:

Proposition 7 *Under Monopolistic Competition: (i) aggregate variable profit is:*

$$\Pi_i^V = \frac{X_i}{\sigma},$$

(ii) aggregate profit is:

$$\Pi_i = \delta^{MC} X_i \tag{5.12}$$

where:

$$\delta^{MC} = \frac{\sigma - 1}{\theta\sigma}$$

and (iii) average profit per producer is:

$$\frac{\sigma - 1}{\theta - (\sigma - 1)} E_i = \frac{\sigma - 1}{\theta - (\sigma - 1)} w_i F_i \tag{5.13}$$

where

$$X_i = \frac{\theta \bar{m}}{\theta \bar{m} - 1} w_i L_i$$

³Ghironi and Melitz (2004) make this point in a related model.

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is total spending and L_i is the total labor force engaged either directly in production or in overhead.

An implication is that:

$$\frac{X_i}{E_i} = \frac{\theta \bar{m}}{\theta \bar{m} - 1} \frac{L_i}{F_i}, \quad (5.14)$$

so that we can write the price index as:

$$P_i = \gamma^{MC} \left(\frac{\theta \bar{m}}{\theta \bar{m} - 1} \cdot \frac{L_i}{\sigma F_i} \right)^{-[\theta - (\sigma - 1)] / [(\sigma - 1)\theta]} \Phi_i^{-1/\theta}$$

giving us an expression for the real wage in terms of fundamentals:

$$\frac{w_i}{P_i} = \frac{T_i^{1/\theta}}{\gamma^{MC}} \left(\frac{\theta \bar{m}}{\theta \bar{m} - 1} \cdot \frac{L_i}{\sigma F_i} \right)^{[\theta - (\sigma - 1)] / [(\sigma - 1)\theta]}.$$

Note that the real wage increases not only with the state of technology T_i , but with the size of the labor force relative to the entry requirement.

Combining (5.11) with (5.14) gives us an expression for the share of overhead workers in the labor force:

$$\frac{H_i F_i}{L_i} = \frac{\theta - (\sigma - 1)}{\theta \sigma - (\sigma - 1)}.$$

A difference between our formulation and the standard model of monopolistic competition is producer heterogeneity. As described in Chapter 4, producers differ in the quality of their techniques for production, with efficiency drawn from the Pareto distribution with parameter θ . In the limit as $\theta \rightarrow \infty$, all producers are the same. Note that, taking this limit, the analysis above reduces to the closed economy version

of monopolistic competition expositied in Chapter 3. In particular, profits net of fixed costs, go to zero as all producers are at the margin of entry. With finite θ , there are rents associated with better techniques.

A 5.4 Cournot Competition

A 5.5 Conclusion

In summary, our assumptions about ideas provide the flexibility to explore a wide range of market structures. The three that we have explored in depth in no way exhaust the possibilities. With Cournot competition, for example, multiple varieties of the same good (with different costs) could compete against each other, even with $\sigma' = 0$, thus potentially bringing in producers with higher costs ($k > 2$). Alternatively, one could admit intermediate values of σ' (some finite value strictly greater than σ), again making $k > 2$ relevant. One could also examine economies with mixed market structures, for example, one in which some goods are supplied monopolistically because of patent protection or trade secrets, while others are supplied competitively.

Two particular aspects of our analysis are key for the following two chapters. One is the determination of profits in Bertrand and monopolistic competition, which will serve as the driving force of innovation in Chapter 7. The other is the price index under each of the different market structures that we consider, which differs only in

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the constant term. The relationship between the state of the economy Φ and the price level is invariant to the form of competition. This invariance allows us to investigate a large number of issues in international trade, the topic of the next chapter, without taking a stand on market structure.

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A **5.6 Appendix**

This Appendix provides proofs of Propositions 1 and 3 through 6 stated in the Chapter.

B **5.6.1 Proof of Proposition 1**

Since $p(j) = C^{(1)}(j)$, if we were to integrate across goods we would calculate:

$$P_i = \left[\int_0^J C^{(1)}(j)^{1-\sigma} dj \right]^{1/(1-\sigma)}.$$

For technical reasons we prefer to integrate across costs, which yields:

$$\begin{aligned} P_i &= \left[J \int_0^\infty c_1^{1-\sigma} dF_1(c_1) \right]^{1/(1-\sigma)} \\ &= J^{1/(1-\sigma)} E \left[(C^{(1)})^{1-\sigma} \right]^{1/(1-\sigma)}. \end{aligned}$$

Setting $b = 1 - \sigma$ in (4.10) delivers the result.

B **5.6.2 Proof of Proposition 3**

Since:

$$P_i^{1-\sigma} = \int_0^J P(j)^{1-\sigma} dj,$$

we integrate across the ratio $M' = C^{(2)}/C^{(1)}$ to get:

$$P_i^{1-\sigma} = J \int_1^{\bar{m}} E \left[(C^{(2)})^{1-\sigma} \right] dF_{2/1}(m') + J \int_{\bar{m}}^\infty E \left[(\bar{m}C^{(2)}/m')^{1-\sigma} \mid C^{(2)}/C^{(1)} = m' \right] dF_{2/1}(m').$$

>From (4.14) the distribution of M' is independent of $C^{(2)}$, so we can write:

$$\begin{aligned} P_i^{1-\sigma} &= JE \left[(C^{(2)})^{1-\sigma} \right] (1 - \bar{m}^{-\theta}) + JE \left[(C^{(2)})^{1-\sigma} \right] \bar{m}^{1-\sigma} \int_{\bar{m}}^{\infty} (m')^{-(1-\sigma)} dF_{2/1}(m') \\ &= JE \left[(C^{(2)})^{1-\sigma} \right] \left[1 + \frac{\sigma - 1}{\theta - (\sigma - 1)} \bar{m}^{-\theta} \right]. \end{aligned}$$

Hence:

$$P_i = J^{1/(1-\sigma)} E \left[(C^{(2)})^{1-\sigma} \right]^{1/(1-\sigma)} \left[1 + \frac{\sigma - 1}{\theta - (\sigma - 1)} \bar{m}^{-\theta} \right]^{1/(1-\sigma)}.$$

The result follows from applying (4.11).

B 5.6.3 Proof of Proposition 4

We rewrite expression (5.8) as:

$$\pi(j) = (1 - M(j)^{-1}) \left(\frac{P(j)}{P_i} \right)^{1-\sigma} X_i$$

Integrating across j , and dividing by total spending, we get:

$$\frac{\Pi_i}{X_i} = 1 - \frac{1}{P_i^{1-\sigma}} \int_0^J M(j)^{-1} P(j)^{1-\sigma} dj.$$

Following closely the proof of Proposition 3:

$$\begin{aligned} & \int_0^J M(j)^{-1} P(j)^{1-\sigma} dj \\ &= J \int_1^{\bar{m}} E \left[(C^{(2)})^{1-\sigma} \right] (m')^{-1} dF_{2/1}(m') + \frac{J}{\bar{m}} \int_{\bar{m}}^{\infty} E \left[(\bar{m}C^{(2)}/m')^{1-\sigma} \right] dF_{2/1}(m') \\ &= JE \left[(C^{(2)})^{1-\sigma} \right] \left[\frac{\theta}{1+\theta} (1 - \bar{m}^{-\theta-1}) + \frac{\theta}{\theta - (\sigma - 1)} \bar{m}^{-\theta-1} \right] \\ &= JE \left[(C^{(2)})^{1-\sigma} \right] \frac{\theta}{1+\theta} \left[1 + \frac{\sigma - 1}{\theta - (\sigma - 1)} \bar{m}^{-\theta} \right] \end{aligned}$$

Dividing by $P^{1-\sigma}$, from (5.7) above, gives the result.

B 5.6.4 Proof of Proposition 5

The variable profit of a firm with cost c and charging price p is:

$$\Pi^V(c) = (p - c)X(j)/p.$$

As is easy to verify, our cost structure preserves a basic result from monopolistic competition, that profit is at a maximum at:

$$p = \bar{m}c$$

so that variable profit is:

$$\Pi^V(c) = \frac{X(j)}{\sigma} = \frac{X_i}{\sigma} \left(\frac{\bar{m}c}{P_i} \right)^{1-\sigma}, \quad (5.15)$$

which decreases in cost c . Hence entry is profitable only for producers with cost $c \leq \bar{c}$ given by:

$$\bar{c}_i = \left(\frac{X_i}{\sigma E_i} \right)^{1/(\sigma-1)} \frac{P_i}{\bar{m}}. \quad (5.16)$$

For this case we can rewrite the price index as the integral over the prices charged by sellers with different costs c in the range $[0, \bar{c}]$ weighted by the measure of suppliers with that cost. This Pareto measure is the derivative of the function (4.8) with respect

to c . The price index is consequently:

$$\begin{aligned}
 P_i &= \left[\int_0^{\bar{c}_i} (\bar{m}c)^{1-\sigma} dH_i(c) \right]^{1/(1-\sigma)} \\
 &= \bar{m} \left[\Phi_i \int_0^{\bar{c}_i} \theta c^{\theta-\sigma} dc \right]^{1/(1-\sigma)} \\
 &= \bar{m} \left[\frac{\theta \Phi_i}{\theta - (\sigma - 1)} \bar{c}_i^{\theta-(\sigma-1)} \right]^{1/(1-\sigma)}.
 \end{aligned} \tag{5.17}$$

Equations (5.16) and (5.17) each involve the price index P_i and the maximum cost for entry \bar{c}_i . Solving for each we get a cut-off cost:

$$\bar{c}_i = \left(\frac{\theta - (\sigma - 1)}{\theta \Phi_i} \frac{X_i}{\sigma E_i} \right)^{1/\theta}$$

Substituting \bar{c} into (5.17) and (4.8) establishes the proposition.

B 5.6.5 Proof of Proposition 6

Using (5.15) above, total variable profit is:

$$\begin{aligned}
 \Pi_i^V &= \frac{X_i}{\sigma} \left(\frac{\bar{m}}{P_i} \right)^{1-\sigma} \int_0^{\bar{c}_i} c^{1-\sigma} dH_i(c) \\
 &= \frac{X_i}{\sigma} \left(\frac{\bar{m}}{P_i} \right)^{1-\sigma} \frac{\theta \Phi_i}{\theta - (\sigma - 1)} \bar{c}_i^{\theta-(\sigma-1)}
 \end{aligned}$$

Substituting in the price index as it appears in (5.17) yields (i). Total entry cost is the individual entry cost E_i multiplied by the measure of entrants (5.11). Subtracting total entry cost $H_i E_i$ from Π_i^V delivers (ii). Dividing Π_i by H_i in (5.11) gives (iii).