Chapter 3

Analytic Foundations

Our analysis in the next chapters draws on several literatures. Our quantitative analysis builds on the "gravity" approach to modeling bilateral trade flows. Our theoretical analysis builds on the theory of trade with monopolistic competition, the Ricardian model of trade with a continuum of goods, and the literature on growth in the global economy. We don't attempt to cover each of these areas in depth, but rather refer the reader to some recent, very thorough, surveys. Instead we present some basic results, first from international trade and then from economic growth, that our work builds upon.

3.1 International Trade

We consider, in turn, the Armington model of trade, the monopolistically-competitive model of trade, and the Ricardian model with a continuum of goods. In our analysis of each of these models, our theme is to introduce trade costs from the start while pointing to general equilibrium implications.

3.1.1 Armington and the Gravity Equation

The Armington model is built on the idea that international trade reflects consumers' desire for foreign goods.[1] Because the force for trade comes from consumer preferences, we can simplify our analysis by ignoring the production of goods altogether. We can, instead, use the Armington model to focus on the role of trade costs in a general equilibrium analysis of international trade. Many of the relationships that arise in this simple model will appear again, in some guise, when we turn to more realistic models.

Following Anderson (1979), we can also use the Armington model to derive the gravity equation (2.1). As shown in the previous chapter, the gravity equation is a good statistical representation of bilateral trade flows. There is something to be said for a theory that is consistent with it.[2] Anderson and van Wincoop (2003) show that a

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[1] This assumption, named for Armington (1969), has been a workhorse in the quantitative analysis of international trade.

[2] Deardorff (1998) provides a nice explanation of how the gravity equation relates to other theories of international trade.

theoretical derivation of the gravity equation can resolve some puzzles that have arisen in interpreting estimates of it. We summarize their arguments below.

Consider N countries. Each country i has a quantity y_i of a good unique to it. We can name this good, after the country it comes from, as "good i". We can think of y_i simply as an endowment. Alternatively, treating input supplies and technology as exogenous, we can think of y_i as the output of a good (or composite of goods) which the country completely specializes in producing. Specialization itself is not modeled, as it is in the monopolistically competitive and Ricardian cases taken up below.

Consumers everywhere have identical constant elasticity of substitution (CES) preferences, with a preference weight $\alpha_i > 0$ on good i. The elasticity of substitution between goods from different countries is $\sigma > 1$. Welfare in country n is thus:

$$U_n = \left[\sum_{i=1}^{N} \alpha_i^{1/\sigma} y_{ni}^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}$$

where y_{ni} is country n's consumption of the good i.

In place of a transport sector, we adopt Samuelson's (1952) "iceberg" assumption. Delivering a unit of i's good to n requires shipping $d_{ni} \geq 1$ (with $d_{ii} = 1$) units from i. Anderson and van Wincoop refer to d_{ni} as the "bilateral resistance" to trade between n and i.

The assumption that trade costs augment production costs multiplicatively is very common in the general equilibrium modeling of international trade. A natural alternative is to treat the transport cost as additive, but a reformulation of the

analysis in this chapter and below under the additive alternative would be vastly more complicated, as the reader embarking on such as task can quickly verify.

Hummels and Skiba (2004) shed some light on how trade costs vary with production cost by regressing freight costs on f.o.b. prices in a set of destinations within a wide range of narrowly defined product categories. They find that freight costs increase with f.o.b. price with an elasticity strictly below one (the elasticity implied by the multiplicative assumption) but well above zero (the elasticity implied by a purely additive specification). There results indicate the need for both more theory and measurement of trade barriers. For one thing, they do not provide evidence on how freight costs vary with f.o.b. prices across product categories. For another, freight costs constitute only one component of the geographic barriers to trade, which also include the cost of searching for a supplier, negotiating a purchase, and servicing the product subsequently.[3]

Consumption around the world of good i is constrained by the world's endow-

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[3]Rauch (1999) provides important indirect evidence on the role of trade barriers that arise for reasons other than shipping. He divides internationally traded goods into three categories: (1) goods for which there are organized exchanges, (2) goods offered for sale at a posted reference price by the supplier, and (3) differentiated products. Estimating gravity equations for goods in different categories, he finds that distance and differences in language are most inhibiting for trade among goods in the third category. His interpretation is that trade in differentiated products requires search and negotiation, which are facilitated by proximity and common language.

ment of it. Taking account of the iceberg transport technology and summing across destinations, the resource constraint for each good i is

$$y_i = \sum_{n=1}^{N} d_{ni} y_{ni}.$$

Due to bilateral resistance, the law of one price will not hold, i.e. the price of good i will differ across markets n. We denote the price of good i in country n by p_{ni}. Taking account of these price differences, country i's total income is

$$Y_i = \sum_{n=1}^{N} p_{ni} y_{ni},$$

of which $Y_i - p_{ii} y_{ii}$ is export income. Country n spends

$$X_n = \sum_{i=1}^{N} p_{ni} y_{ni},$$

of which $X_n - p_{nn} y_{nn}$ is spent on imports.

We consider a competitive equilibrium. In particular, we look for a set of prices p_{ni} and consumptions y_{ni} such that: (i) given prices, each country i sells its endowment so as to maximize its income Y_i subject to the resource constraint and (ii) given income Y_n and prices, the representative consumer in each country n allocates spending across goods i so as to maximize utility U_n subject to the budget constraint $X_n \leq Y_n$.

The solution to the consumer's problem is to spend $X_n = Y_n$ with expenditure on good i, $X_{ni} = p_{ni} y_{ni}$, given by

$$X_{ni} = \alpha_i \left(\frac{p_{ni}}{P_n} \right)^{-(\sigma-1)} X_n, \tag{3.1}$$

where P_n is the CES price index in country n:

$$P_n = \left[\sum_{k=1}^{N} \alpha_k (p_{nk})^{-(\sigma-1)} \right]^{-1/(\sigma-1)}. \tag{3.2}$$

This solution is standard except that we express it in terms of expenditures (rather than quantities) to make a more explicit link to the data.[4]

The solution for international price differences is very simple. At any finite prices, each country n demands some of good i. Country i will be willing to sell positive amounts to each country n only if p_{ni}/d_{ni} is the same across markets. Thus, the competitive equilibrium implies $p_{ni}/d_{ni} = p_{ii}$, or equivalently, $p_{ni} = d_{ni}p_{ii}$ for all n.

Since a similar expression keeps coming up in different models, we assemble

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[4]The representative consumer in n chooses y_{ni} $(i = 1, \ldots, N)$ so as to maximize U_n given prices p_{ni} $(i = 1, \ldots, N)$ and income Y_n, and subject to the budget constraint $X_n = \sum_{i=1}^{N} p_{ni}y_{ni} \leq Y_n$. The first-order conditions (FOC's) can be written:

$$\lambda_n p_{ni} y_{ni} = U_n^{1/\sigma} \alpha_i^{1/\sigma} y_{ni}^{(\sigma-1)/\sigma},$$

where λ_n is the lagrangian multiplier on the budget constraint. Taking the ratio of the FOC's for purchases from countries k and i yields

$$\frac{X_{nk}}{X_{ni}} = \frac{\alpha_k}{\alpha_i} \left(\frac{p_{nk}}{p_{ni}} \right)^{-(\sigma-1)}.$$

Summing both sides of this expression over $k = 1, \ldots, N$

$$\frac{X_n}{X_{ni}} = \frac{\sum_{k=1}^{N} \alpha_k (p_{nk})^{-(\sigma-1)}}{\alpha_i (p_{ni})^{-(\sigma-1)}},$$

which can be rearranged to get (3.1) with the price index (3.2).

what we have so far into an expression for the fraction of spending from n devoted to goods from i:

$$\frac{X_{ni}}{X_n} = \frac{\alpha_i \left(p_{ii}d_{ni}\right)^{-(\sigma-1)}}{\sum_{k=1}^{N} \alpha_k (p_{kk}d_{nk})^{-(\sigma-1)}} \tag{3.3}$$

Country i's trade share in n is its contribution to the sum $\sum_{k=1}^{N} \alpha_k (p_{kk}d_{nk})^{-(\sigma-1)}$. Its contribution reflects (i) its importance in preferences α_i, (ii) the local price of its goods p_{ii}, and (iii) the cost of getting the goods from i to n, as reflected by d_{ni}. Note how the elasticity of substitution governs the sensitivity of trade shares to trade costs.

It remains only to solve for the local price of country i's good, p_{ii}, for $i = 1, ..., N$. To do so, multiply both sides of the resource constraint by p_{ii} and apply the result about international price differences to obtain

$$p_{ii}y_i = \sum_{n=1}^{N} p_{ni}y_{ni} = Y_i.$$

This equation states that country i's income is simply the value of its endowment at local prices or, equivalently, its sales around the world. Subtracting $p_{ii}y_{ii}$ from both sides, we get a trade balance equation that exports equal imports. Writing the income equation in terms of primitives and unknowns:

$$p_{ii}y_i = \sum_{n=1}^{N} \frac{\alpha_i \left(p_{ii}d_{ni}\right)^{-(\sigma-1)}}{\sum_{k=1}^{N} \alpha_k (p_{kk}d_{nk})^{-(\sigma-1)}} p_{nn}y_n \tag{3.4}$$

These equations, one for each country i (with, by Walras Law, one redundant) can be solved for the $N - 1$ local output prices, p_{ii} (with $p_{NN} = 1$ serving as the numeraire).

Except for a few special cases, discussed below, there are not analytical solutions to these equations. For most cases they must be solved numerically.

As discussed above, we can easily reinterpret this model as one in which each country i has an endowment L_i of labor specialized in the production of the country's distinct good. With output per worker a_i we replace y_i with $a_i L_i$ and p_{ii} with w_i/a_i, where w_i is the wage.

C   Gravity Results

Deriving the gravity equation side-steps having to solve equations (3.4) explicitly for local output prices. To get an expression that is closer to the standard gravity formulation we rewrite the equations as:

$$Y_i = \alpha_i p_{ii}^{1-\sigma} \sum_{n=1}^{N} \left(\frac{d_{ni}}{P_n}\right)^{-(\sigma-1)} X_n. \tag{3.5}$$

Solving (3.5) for $\alpha_i p_{ii}^{1-\sigma}$ and substituting into (3.1), with $p_{ni} = d_{ni} p_{ii}$, gives:

$$X_{ni} = \frac{Y_i X_n}{\Xi_i} \left(\frac{d_{ni}}{P_n}\right)^{-(\sigma-1)}, \tag{3.6}$$

where:

$$\Xi_i = \sum_{m=1}^{N} \left(\frac{d_{mi}}{P_m}\right)^{-(\sigma-1)} X_m.$$

Setting $\tau_{ni} = d_{ni}^{\sigma-1}$ in (3.6) yields an expression with all the ingredients of the simple gravity equation (2.1).[5]

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[5] Anderson and van Wincoop go on to show that if trade costs are symmetric, meaning that $d_{ni} = d_{in}$, then $\Xi_i = P_i^{1-\sigma}$. Countries that have lower prices also have greater market potential (by virtue

But, as Anderson and van Wincoop point out, equation (3.6) has two additional terms that reflect the proximity of third countries. One is the price index P_n in the destination. Given its own cost of shipping to a destination, country i will fare better in countries that are more remote from other suppliers, since i faces less competition there. A destination's remoteness is reflected in a high price index there. The other term is Ξ_i, often called the source's "market potential." If source i is itself more remote from other countries, as implied by a smaller value of Ξ_i, it will sell more in market n given its cost of shipping there. Anderson and van Wincoop refer to the effects of P_n and Ξ_i as reflecting country i's and n's "multilateral resistance" to trade. A high Ξ_i means that i has good selling opportunities outside market n, so will, other things equal, sell less to n. A lower P_n means that country n has good buying opportunities elsewhere than from i, so, other things equal, will buy less from i.

Anderson and van Wincoop point out that larger countries will likely have both higher Ξ_i and lower P_i, reducing their bilateral trade, but will also tend to be farther from their neighbors. Estimating the standard formulation of the gravity equation with the these multilateral resistance terms omitted then yields estimates that overstate the effect of distance on bilateral resistance to trade. By the same reasoning, the

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of their proximity to other countries both as suppliers and as consumers). In this case the gravity equation becomes:

$$X_{ni} = Y_i X_n \left(\frac{d_{ni}}{P_i P_n} \right)^{1-\sigma}.$$

standard formulation with multilateral resistance terms omitted would understate the effect of size on trade. As we saw in the previous chapter, the raw elasticity of imports with respect to absorption is less than one.

Gravity is a natural explanation for the dampening effect of distance on trade so evident in Figures 4 and 5 from the previous chapter. Note that our formulation of the vertical axis in these figures implies that we are relating distance to (the square root of) the statistic $X_{ni}X_{in}/X_{nn}X_{ii}$. Equation (3.1), gives:

$$\frac{X_{ni}X_{in}}{X_{nn}X_{ii}} = \left(d_{ni}d_{in}\right)^{-(\sigma-1)}.$$

That is, our normalization purges the theoretical gravity relationship of anything but its strictly bilateral component, including multilateral resistance, thus addressing the Anderson-van Wincoop critique.[6]

C   General Equilibrium Results

While the gravity equation allows us to side-step solving (3.4), it does not allow us to address questions about the gains from trade. To address those issues, note that the Armington model implies

$$U_n = \frac{p_{nn}y_n}{P_n}.$$

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[6]Suppose we posit that trade costs are related to the distance k_{ni} between n and i according to $d_{ni} = d_{in} = \alpha(k_{ni})^{\beta}$ for $n \neq i$. Our findings in Figures 4 and 5 suggest that $(\sigma-1)\beta \approx 1$ has remained constant while, over time, α has fallen, leading to the rise in international trade.

The welfare of country n is increasing in the quantity of its physical endowment y_n and it's terms of trade p_{nn}, while welfare is decreasing in the price level P_n.[7]

For a simple back-of-the-envelope calculation of gains from trade, we can simply evaluate (3.1) for $i = n$. Solving the result for the terms of trade relative to the price level p_{nn}/P_n, and multiplying by the endowment, yields:

$$U_n = \left[\frac{\alpha_n}{(X_{nn}/X_n)} \right]^{1/(\sigma-1)} y_n.$$

Given its endowment, country n is better off if, in equilibrium, a large share of its expenditure is devoted to imports. In autarky, of course, this share is zero and welfare is $\alpha_n^{1/(\sigma-1)} y_n$. Thus the welfare gain from trade, relative to autarky, is $(X_{nn}/X_n)^{-1/(\sigma-1)}$. This approach is limited for counterfactual analysis, however, since it does not tell us how X_{nn}/X_n changes with the underlying parameters.

In a couple of special cases we can solve equations (3.4) explicitly. The first

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[7] The theortical price index P_n is the deflator that converts expenditures X_n into utility and $X_n = Y_n = p_{nn}y_n$. More formally, start with the FOC from the consumer's problem given in the footnote above, which can be rewritten as:

$$\alpha_i^{1/\sigma} y_{ni}^{(\sigma-1)/\sigma} = \lambda_n X_{ni} U_n^{-1/\sigma}.$$

Plugging the FOC into the utility function yields $U_n = \lambda_n X_n$. Substituting this result back into the FOC and rearranging gives:

$$X_{ni} = \alpha_i X_n \left(\lambda_n p_{ni} \right)^{1-\sigma}.$$

Thus, $\lambda_n = 1/P_n$ and hence $U_n = X_n/P_n$.

case is when $d_{ni} = 1$ for all n and i. We then obtain $p_{nn} = [(\alpha_n/y_n)/(\alpha_N/y_N)]^{1/\sigma}$. Notice that the terms of trade turn against a country with a larger endowment. Plugging this result into the equation for the price index delivers:

$$U_n = \alpha_n^{1/\sigma} y_n \left[\sum_{i=1}^{N} \alpha_i^{1/\sigma} (y_i/y_n)^{(\sigma-1)/\sigma}\right]^{1/(\sigma-1)}.$$

The other special case is if countries are symmetric with $d_{ni} = d$, for all $n \neq i$. In that case, since $p_{ii} = p$ and $P_n = P$, equation (3.4) can be solved for p/P to yield:

$$U = \alpha^{1/(\sigma-1)} \left[1 + (N-1)d^{-(\sigma-1)}\right]^{1/(\sigma-1)} y$$

Note that welfare is increasing in the number of countries and decreasing in the trade cost.

The Armington assumption allows us to focus purely on the role of trade costs without having to model the forces that shape specialization. But it removes from consideration a host of issues of interest. Gains from trade occur purely through exchange, rather than through specialization in production. Given that much of the policy interest in trade concerns exactly issues of industrial structure, we now turn to theoretical frameworks in which trade has nontrivial implications for who makes what.

3.1.2   Monopolistic Competition

The theory of monopolistic competition, developed in its modern form by Dixit and Stiglitz (1977), has provided a more fertile ground for the quantitative analysis of

trade flows. The theoretical implications of the framework for international trade were explored in a series of papers by Krugman (1979a, 1980) and, most thoroughly, in a book by Helpman and Krugman (1985). As we see below, monopolistic competition delivers a formulation for bilateral trade flows that mirrors that implied by the simpler Armington assumption.

In its simplest version the model does not focus on differences in factor intensity, so we can posit a single factor of production, which we call labor. Each country i has a given endowment L_i.[8] Workers are free to engage in different activities at home, but don't move between countries. The wage w_i is thus the same across all activities in i, but can differ between countries. [9]

In its original formulation, differences in efficiency across goods and countries were not a focus, so one can posit a common output per worker z. Setting up the production of a good requires an additional F_i workers, where, again, in the simplest formulation there is no difference across countries. The range of goods produced

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[8]More generally, we could posit a composite input bundle, but differences in intensities across individual inputs would not play a role in trade and specialiation.

[9]Throughout this chapter we treat products as final goods. Essentially equivalent results emerge if products are instead intermediates used to produce a nontraded final good according to a CES production function, as demonstrated by Ethier (1979) for monopolistic competition. We will not continue to point out this alternative interpretation, but ask the reader to remain aware that with appropriate redefinitions the goods in question could be final, intermediate, or both. Intermediate goods will emerge in subsequent chapters.

and consumed in a country arises endogenously through entry into production. Each producer makes a different good. The space of goods is most easily modelled as a continuum. We index goods by j.

A representative consumer has preferences of the form:

$$U = \left[\int y(j)^{(\sigma-1)/\sigma} dj \right]^{\sigma/(\sigma-1)}$$

where $y(j)$ is consumption of good j and $\sigma > 1$ is the elasticity of substitution. (where $y(j) = 0$ if j is unavailable.) If total spending in country n is X_n, spending on commodity j in country n is:

$$x_n(j) = X_n \left(\frac{p_n(j)}{P_n} \right)^{1-\sigma}$$

where $p_n(j)$ is the price of good j in country n, and P_n is the price index:

$$P_n = \left[\int p_n(k)^{1-\sigma} dk \right]^{1/(1-\sigma)}.$$

(We can think of a good that isn't available in country n as having an infinite price.)

The market structure is monopolistic competition. Each good is produced by a separate monopolist who takes total spending X_n and the price index P_n in each market as given. Markets are segmented so that producers can set a different price in each national market. Profit maximization results in a price markup over unit cost, inclusive of transport, of $\overline{m} = \sigma/(\sigma-1)$. Thus a firm in i will charge a price $p = \overline{m} w_i d_{ni}/z$ when selling in n. We choose units so that $z = 1$. Hence the revenue of

a representative firm from i in n is:

$$x_{ni} = X_n \left(\frac{\overline{m} w_i d_{ni}}{P_n} \right)^{1-\sigma} \tag{3.7}$$

while its profit in market n is:

$$\pi_{ni} = \frac{x_{ni}}{\sigma}. \tag{3.8}$$

Denoting the measure of goods produced in i as H_i, the price index in market n is:

$$P_n = \overline{m} \left[\sum_{i=1}^{N} H_i (w_i d_{ni})^{1-\sigma} \right]^{1/(1-\sigma)}.$$

In the basic formulation the fixed cost $w_i F_i$ applies to a firm in i establishing a product, not to entering a market. Since free entry eliminates profit, all income goes to labor (either directly in production or for setting up firms) so that $X_n = w_n L_n$.

Two conditions determine the vector of wages and the measure of products produced in each country. One is the zero profit condition enforced by free entry, which establishes that:

$$w_i^{\sigma} \sigma F_i = \Xi_i \quad i = 1, ..., N, \tag{3.9}$$

where

$$\Xi_i = \sum_{n=1}^{N} X_n \left(\frac{d_{ni}}{P_n} \right)^{1-\sigma}$$

is equivalent to the "market potential" term for the Armington case. The other is that total spending on country i's production equal its wage bill, which establishes that:

$$w_i L_i = H_i \sum_{n=1}^{N} x_{ni}$$

which, using (3.7), can be written:

$$w_i^\sigma L_i = H_i \Xi_i \quad i = 1, ..., N \tag{3.10}$$

An immediate implication of combining these two conditions is that the measure of products a country produces is proportional to its labor force, specifically, that:

$$H_i = \frac{L_i}{\sigma F_i}. \tag{3.11}$$

Note in particular that even though H_i is endogenous, it does not depend on the extent of trade barriers. In particular, trade does not reduce the measure of goods that a country produces, as it can in the Ricardian model taken up next.

Relative wages are given by the solution to the system of equations:

$$w_i^\sigma = \frac{\Xi_i}{\sigma F_i} = \sum_{n=1}^{N} \frac{w_n L_n d_{ni}^{1-\sigma}}{\sum_{k=1}^{N} L_k (w_k d_{nk})^{1-\sigma}} \quad i = 1, ..., N. \tag{3.12}$$

Note the role for geography in determining wages. Countries with lower market potential, i.e., more distant from large markets, need to have lower relative wages in order to compete abroad.

Like the Armington model developed above, monopolistic competition readily yields an expression for bilateral trade. The value of exports from i to n is:

$$X_{ni} = H_i x_{ni} = \frac{H_i (\overline{m} w_i d_{ni})^{1-\sigma}}{P_n^{1-\sigma}} X_n.$$

Analogous with equation (3.3) for Armington model, we have the following expression

for the fraction of n's expenditure devoted to goods from i:

$$\frac{X_{ni}}{X_n} = \frac{L_i(w_i d_{ni})^{1-\sigma}}{\sum_{k=1}^{N} L_k(w_k d_{nk})^{1-\sigma}} \tag{3.13}$$

The labor forces L_i replace the preference terms α_i in the Armington model, while wages replace the local prices p_{ii}. Otherwise the expression is the same. The major difference is that under the Armington assumption the share of a country's goods in preferences is exogenous while, under monopolistic competition, it rises with the labor force (since larger countries endogenously produce a greater variety of distinct goods).[10]

To get an expression more in line with the gravity equation ,we use (3.10) to write:

$$Y_i = w_i L_i = w_i^{1-\sigma} H_i \Xi_i.$$

Substituting the solution for $w_i^{1-\sigma} H_i$ into the relationship above gives:

$$X_{ni} = \frac{Y_i X_n}{\Xi_i} \left(\frac{d_{ni}}{P_n}\right)^{1-\sigma}, \tag{3.14}$$

exactly the same expression yielded by the Armington analysis, expression (3.6). An-

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[10]Interpreting the Armington model as one with specialized production, an important difference with monoplistic competition is the implication of having relatively more labor. Given the preference terms α_i in the Armington model, having more workers is hurtful since it worsens the terms of trade. In monopolistic competition more workers at home is good, as it means that a greater variety of products can be purchased without having to incur trade costs. (Once trade costs disappear, relative size is a matter of indifference). Fixing relative labor supplies, a bigger world is a matter of indifference in Armington, but a good thing with monopolistic competition.

derson and van Wincoop's message about the importance of including multilateral resistance stands.

But the monopolistic competition framework has quite different implications for how aggregate trade volumes vary at the extensive margin (number of goods shipped) and the intensive margin (amount of each good shipped). In Armington, all variation is at the extensive margin: A larger country exports more because it exports more of a given good, while it also imports more of each of a given set of goods. Under monopolistic competition, a larger country exports more because it exports a greater variety. Hence size-induced variation in export volumes across countries is purely at the extensive margin. But, as in Armington, size-induced variation in import volumes is purely at the intensive margin. Since every country imports every good, a larger country imports more purely because it is buying more of each one.

Monopolistic competition has provided the basis of a number of empirical studies of bilateral trade patterns, based on variants of equation (3.14). An early example is Helpman (1987), followed by many others.[11] Closest to our own work is Redding and Venables (2004). Their analysis also includes a role for intermediate inputs, which also play an important part in our own work, as we discuss in Chapter 6.

An additional feature of monopolistic competition as a framework for analyzing trade flows is that it identifies a nontrivial role for individual producers. As data are

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[11]Notable contributions include Hummels and Levinsohn (1995) and Debaere (2005).

becoming much more available on plant and firm participation in international trade, this feature is a major plus.[12] The basic framework makes stark predictions about how bilateral trade flows break down into the number of producers selling and how much each one sells. Looking across exporting countries, large countries export more because they have proportionately more firms, but an individual firm from a large country is not predicted to sell more abroad than one from a small country. Differences in export volumes per firm are dictated by geography rather than by country size. Looking across importing countries, large countries buy more because they purchase proportionately more from each foreign producer. All destinations purchase from the full range of individual producers.

Figure 7 of the previous chapter shows that in this last prediction the basic model falls flat on its face in two respects: Large markets attract systematically more firms, while differences in an export country's market share across destinations are almost all due to the number of its firms that sell there.[13]

Melitz (2003) provides an important extension to the monopolistically com-

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[12]In contrast, trade theories based on perfect competition, such as the Ricardian one we turn to next, make no prediction about what to expect at the level of the individual producer to guide the analysis of the data.

[13]Hummels and Klenow (2005), using data on detailed product categories, which may proxy for the number of individual producers, look at the export breakdown. They find that the elasticity of the number of varities exported with respect to size is about .6; large, but less than the 1 predicted by the basic model.

petitive model of trade which can potentially loosen its tight implications about the margin of entry. He introduces two innovations. First, he assumes that there is heterogeneity across potential producers in the unit cost of production. Second, as in Romer (1994), he posits a fixed cost of entering a foreign market. These assumptions have implications for variation in trade volumes at the extensive and intensive margins. More recently, Helpman, Melitz and Rubinstein (2004) have adapted his approach to the quantitative analysis of bilateral trade flows. In particular, unlike most theoretical formulations of the gravity model their analysis allows for observations of zero trade. Since this analysis is intimately connected with our own in the following chapters, we postpone further discussion of these contributions for later chapters.

Unlike the Armington approach in which each country produces a different set of goods for exogenous reasons, under monopolistic competition producers in each country endogenously choose to produce a different set of goods. But the model does not deliver implications for how trade might shift specialization across industries.

3.1.3 Ricardo with a Continuum of Goods

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Ricardo (1821) provided a model of the effects of trade on specialization in general equilibrium. To restate the canonical example, two countries (say H and F) have endowments of labor and constant-returns-to-scale technologies for producing each of two commodities (say C and W) using only labor. We can describe these technologies in

terms of output per worker $z_i(j)$, $i = H, F$, and $j = C, W$. Workers are perfectly mobile between activities within a country, but not between countries. Goods are costlessly traded.

Ricardo showed that if country H has a comparative advantage in good C:

$$\frac{z_H(C)}{z_F(C)} > \frac{z_H(W)}{z_F(W)} \tag{3.15}$$

then H exports C and imports W, and at least one of the countries is better off (and neither worse off) due to this trade. Details to be worked out were if the equilibrium involved country H producing only C, country F producing only W, or both.[14]

For nearly two centuries Ricardo's formulation has served as an extremely useful vehicle for illustrating the gains from trade and specialization. Until very recently, however, it did not provide a basis for the quantitative analysis of bilateral trade flows.[15] The impediment is the vast array of possible types of equilibria it throws out (depending on who is completely specialized, or not, in what) in a realistic multicountry, multigood setting.[16].

Something of a breakthrough occurred with Dornbusch, Fischer, and Samuel-

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[14] As Chipman (1965) documents, working out these details took almost a century.

[15] A literature initiated by MacDougall (1951,1952) looked at the relationship between measured productivity across industries and export specialization. This approach was limited to considering a pair of countries as exporters to the rest of the world, however, so could not deal with the simultaneous determination of bilateral trade flows around the world.

[16] Relaxing twoness either in the number of countries or in the number of goods is relatively straighforward, as shown by Jones (1961). It's relaxing both together that causes trouble. Jones provides the

son (1977, henceforth DFS). By treating the space of goods as a continuum, it was no longer necessary to consider separately outcomes with complete and incomplete specialization. The pivotal good that might potentially be produced in both countries has zero measure, so can be ignored.

Since a special case of the DFS model constitutes a special case of our own formulation of trade, we present a synopsis of their model in terms of elements of the approach that we develop in the subsequent chapters.

Consider a unit continuum of goods indexed by $j \in [0,1]$ which can be produced in the home country (H) or the foreign country (F). A worker in country $i = H, F$ can produce $z_i(j)$ units of good j. DFS perform their analysis using the ratio of efficiencies in F and H, defined as:

$$A(j) = \frac{z_F(j)}{z_H(j)}$$

where goods are labelled so that, for any j and j' such that $j' \geq j$, $A(j') \geq A(j)$. Hence the function $A(j)$ is increasing in j. DFS impose the additional requirement that A is continuous and strictly increasing in j.

Preferences are Cobb-Douglas and identical in each country, with each good

criterion for the efficient assignment of goods to countries in higher dimensions, showing that the way to generalize Ricardo's criterion for the assignment of goods to countries is to reformulate inequality (3.15) as the assignment that maximizes the product of labor productivities. But with many countries and goods, even once this assignment is found there are many possible patterns of complete and incomplete specialization that have to be considered to solve for the equilibrium.

j having equal share. Each country i has an endowment of L_i workers. Perfect competition prevails.

Each country i has a wage w_i. Iceberg costs are d_{ni}. The cost of good j in market n if purchased from country i is $w_i d_{ni}/z_i(j)$. Using H's labor as numeraire we set $w_H = 1$. Since goods are bought from the lowest cost source, the foreign country will produce for itself the range of goods $[\underline{j}, 1]$ where $\underline{j}$ satisfies:

$$A(\underline{j}) = \frac{w_F}{d_{FH}}. \tag{3.16}$$

Similarly, the home country will produce for itself the range of goods $[0, \overline{j}]$, where $\overline{j}$ satisfies:

$$A(\overline{j}) = w_F d_{HF}. \tag{3.17}$$

Since $A(j)$ is a strictly increasing function, $\underline{j}$ and $\overline{j}$ are increasing in w_F, with $\underline{j} < \overline{j}$. Income and expenditure in each country n is $w_n L_n$. Thus H's sales at home are just $\overline{j} w_H L_H$ while its export revenues are $\underline{j} w_F L_F$. Full employment in H thus requires that:

$$L_H = \underline{j} w_F L_F + \overline{j} L_H. \tag{3.18}$$

Together, equations (3.16), (3.17), and (3.18) determine $\underline{j}$, $\overline{j}$, and w_F. The solution is unique since the right hand side of the expression:

$$L_H = A^{-1} \left(\frac{w_F}{d_{FH}} \right) w_F L_F + A^{-1} \left(w_F d_{HF} \right) L_H \tag{3.19}$$

is strictly increasing in w_F while the left-hand side is independent of w_F.

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DFS's Ricardian analysis, unlike Armington or monopolistic competition, captures how trade can alter the set of goods a country produces. In the absence of trade $(d_{FH} = d_{HF} = \infty)$ the solution is $\underline{j} = 0$, $\overline{j} = 1$, of course, and we can normalize $w_F = 1$. Each country produces each good on the unit interval. Lowering the iceberg trade cost leads countries to cease production of the goods in which they have strongest comparative disadvantage, goods $j \in [\overline{j}, 1]$ for H and goods $j \in [0, \underline{j}]$ for F, specializing in the goods in which they have a strong comparative advantage, and exporting those in which their comparative advantage is strongest. But, with positive trade costs (and given that $A(j)$ is a continuous function) the world will not be one of perfect specialization. A middle range of goods $j \in [\underline{j}, \overline{j}]$ countries produce for themselves and do not trade.

A shortcoming of the DFS approach as a framework for quantitative analysis is its limitation to two countries. Wilson (1980) provides an important conceptual generalization of DFS to many countries. Like DFS, Wilson represents technologies in each country i as a function $z_i(j)$ defined over $j \in [0, 1]$. Rather than working with ratios of efficiencies, however, his analysis uses the $z_i(j)$ functions directly. Note that one is allowed an overall reordering of the goods to obtain well behaved $z_i(j)$ functions, but the ordering of goods must be common to all countries $i = 1, \ldots, N$. While Wilson's analysis provides a number of comparative static results, it remains to be shown whether there is a parameterization of the functions $z_i(j)$ that makes it

amenable to the quantitative analysis of trade flows between many countries.

In Chapter 6 we take a different approach to extending the Ricardian model to an arbitrary number of countries. The key idea is to treat each $z_i(j)$ as the independent realization of a random variable Z_i that has the same distribution $F_i(z)$ for each good j in country i. This approach allows us to sidestep any characterization of how $z_i(j)$ varies with j in each country i. Rather, given factor costs and trade barriers, we can analyze trade in each good j in terms of the realizations of Z_i for that particular j. Properties of the distributions of Z_i then determine the likelihood that any particular country i will produce and export to destination n. With a continuum of goods, this likelihood will then be the fraction of goods that n buys from i. No particular ordering of the j is required. Here we show how, when applied to the two country case, our approach delivers the DFS framework with a particular form of the function $A(j)$ and the Wilson framework with particular functional forms for the functions $z_H(j)$ and $z_F(j)$.

We assume that the random variable Z_i has the particular distribution:

$$F_i(z) = \Pr[Z_i \leq z] = \exp[-T_i z^{-\theta}]. \tag{3.20}$$

and is independent across countries.[17] The parameter $T_i > 0$ governs country i's overall level of efficiency (absolute advantage) while $\theta > 1$ (common across countries) governs variation in productivity across different goods (comparative advantage). A higher value of T_i means that country i has, on average, higher efficiency draws, while a higher

---

[17]This distribution is called the Type II extreme value (or Fréchet) distribution.

θ means draws are less dispersed.

Our approach proceeds under the convention that, due to the independence of the efficiency draws across j in any country i, probability distribution of the efficiency for any particular good j is also the distribution of efficiency draws across goods. Hence, from (3.20), for each country j we can order goods so that:

$$z_i(j) = \left(-\frac{T_i}{\ln j} \right)^{-1/\theta}.$$

In contrast to Wilson, however, each country requires a different ordering of goods $j \in [0, 1]$.

To derive the $A(j)$ function of DFS from this distribution, think of $j \in [0, 1]$ as the probability that the relative productivity of F to H is less than A. That is:

$$
\begin{aligned}
j &= \Pr \left[\frac{Z_F}{Z_H} \leq A \right] = \Pr \left[Z_F \leq A Z_H \right] \\[2mm]
&= \int_0^\infty \exp \left[-T_F (A z_H)^{-\theta} \right] dF_H(z_H) \\[2mm]
&= \int_0^\infty \exp \left[-T_F (A z_H)^{-\theta} \right] \theta z_H^{-\theta-1} T_H \exp[-T_H z_h^{-\theta}] dz_H \\[2mm]
&= \int_0^\infty \exp \left[-(T_F A^{-\theta} + T_H) z_H^{-\theta} \right] \theta z_H^{-\theta-1} T_H dz_H \\[2mm]
&= \frac{T_H}{T_H + T_F A^{-\theta}} \int_0^\infty \exp \left[-(T_F A^{-\theta} + T_H) z_H^{-\theta} \right] \theta z_H^{-\theta-1} \left(T_H + T_F A^{-\theta} \right) dz_H \\[2mm]
&= \frac{T_H}{T_H + T_F A^{-\theta}}.
\end{aligned}
$$

The last substitution follows from the fact that the integral in the second to the last

expression is over the entire range of the density of the distribution given in (3.20),

with $T_i = T_H + T_F A^{-\theta}$, so has value 1.

Solving for A as a function of j:

$$A(j) = \left(\frac{T_F}{T_H} \frac{j}{1-j} \right)^{1/\theta},$$

which is the equation for an ogee curve. We can now proceed as in DFS, with $A(j)$

taking this particular form, to derive trade patterns and the relative wage.

Since H's expenditure on imports is $(1 - \bar{j})L_H$, we can write:

$$X_{HF} = \frac{T_F (w_F d_{HF})^{-\theta}}{T_H + T_F (w_F d_{HF})^{-\theta}} L_H.$$

This expression looks very similar to those leading to the gravity equation from the

Armington and monopolistic competition models, except that the parameter θ, which

governs variability in productivity, has replaced the parameter σ from the Dixit-Stiglitz

preferences.

For the special case in which $d_{HF} = d_{FH} = 1$, substituting this expression

into (3.19) gives:

$$w_F = \left(\frac{T_F/L_F}{T_H/L_H} \right)^{1/(1+\theta)}. \tag{3.21}$$

Since prices are the same in both countries, w_F measures welfare in the F relative to

H. It is increasing in foreign's overall level of technology T_F relative to its labor force.

With trade costs, we need to account for price differences between countries.

Here it is useful to recaste the model in terms of Wilson's $z_i(j)$ functions. In the two

country case a single ordering of goods suffices if we break $A(j) = z_F(j)/z_H(j)$ into its

components $z_F(j)$ and $z_H(j)$ defined as:

$$z_F(j) = T_F^{1/\theta}(1-j)^{-1/\theta}$$

$$z_H(j) = T_H^{1/\theta}j^{-1/\theta}.$$

In the home country prices are:

$$p_H(j) = 1/z_H(j) \quad j \in [0,\bar{j}]$$

$$p_H(j) = w_F d_{HF}/z_F(j) \quad j \in [\bar{j}, 1]$$

Given Cobb-Douglas preferences, the appropriate measure of the price level in the home

country is

$$P_H = \exp\left\{\int_0^1 \ln p_H(j)dj\right\}.$$

It is easy to verify that we can solve for P_H by simply looking at the average price of

home-produced goods (which, in this specification, equals the average price of imported

goods):

$$
\begin{aligned}
P_H &= \exp\left\{ \frac{-1}{\bar{j}} \int_0^{\bar{j}} \ln z_H(j)dj \right\} \\[2mm]
&= \exp\left\{ \frac{-1}{\bar{j}\theta} \int_0^{\bar{j}} \left[\ln T_H - \ln j\right] dj \right\} \\[2mm]
&= T_H^{-1/\theta} \exp\left\{ \frac{-1}{\bar{j}\theta} \int_{-\ln\bar{j}}^{\infty} xe^{-x}dx \right\} \\[2mm]
&= e^{-1/\theta}(T_H/\bar{j})^{-1/\theta} \\[2mm]
&= e^{-1/\theta}\left[T_H + T_F(w_F d_{HF})^{-\theta} \right]^{-1/\theta}.
\end{aligned}
$$

Notice that the overall level of productivity in each country, T_H and T_F, contribute to a lower price level in the home country.

The special case of symmetry delivers a particularly simple and easy to interpret equilibrium. Setting $T_H = T_F = T$, $L_H = L_F = L$, and $d_{HF} = d_{FH} = d$ we get $w_F = 1$. Although individual prices will differ between countries, the price level will be the same $P_H = P_F = P$. As trade costs d fall, the gains from trade show up via a decline in the price level:

$$
P = e^{-1/\theta}\left[T(1 + d^{-\theta}) \right]^{-1/\theta}.
$$

In the case of no trade barriers, $d = 1$ while, under autarky, $d \to \infty$. In the symmetric case, then, a movement from autarky to costless trade is equivalent to a doubling of the technology parameter T. The intuition is that the country with costless trade has

two efficiency "draws" for each good while on its own it would have only one.

B
3.1.4   Summary for what Follows

How do these various approaches to international trade relate to the data discussed in the previous chapter and to the analysis in the rest of the book? The the Armington model and monopolistic competition yield equations for bilateral trade very much in keeping with observations on gravity. Moreover, as general equilibrium systems they provide a means of connecting these observations with prices and welfare. But these two approaches have limitations.

One is that they have little to say about specialization in production, which has been a central issue in international trade. In Armington there is either no production at all or else countries are assumed, for exogenous reasons, to specialize in nonoverlapping commodities. In monopolistic competition every producer selects a different product from a menu of infinite length, so complete specialization is an endogenous outcome. In neither case is there direct competition between producers of the same or similar commodities. But such competition is at the heart, for example, of trade disputes involving particular industries, such as textiles or aircraft.

Another limitation is the absence of any connection between aggregate measures of international trade and observations on individual producers. While the firm does make an appearance in monopolistic competition, the basic framework cannot

account for the heterogeneity we observe across individual producers described in the previous chapter.

In contrast, the Ricardian model does model international competition and specialization at the level of individual industries. But it has trouble grappling with the high dimensionality of the bilateral trade data. Moreover, it does no better at coming to terms with observations on individual producers.

Chapter 6 we develop a model of international trade that encompasses both the Ricardian model and monopolistic competition. The model is able simultaneously to account for observations on gravity while accommodating the facts on producer heterogeneity in size, productivity, and export participation.

A
3.2 Economic Growth

The Ricardian and monopolistically competitive models of trade posit a given set of technologies available to different countries. How these technologies evolve over time is not addressed. During the 1980s papers by Romer (1986) and Lucas (1988), endogenizing technical change, spawned a large literature, some of which was aimed at understanding growth in a multicountry context. An important precursor to this literature does not endogenize the process of innovation itself, but shows how, together, the processes of innovation and diffusion can generate a common world growth rate, with countries remaining at different relative income levels.

B   3.2.1   A Product-Cycle Model

Krugman (1979b) provides a simple two-country formulation combining Dixit-Stiglitz

preferences with Ricardian specialization. The measure of varieties available in each

country i is fixed at any moment (as in Ricardo) but evolves over time. Competition is

perfect and there are no transport costs. Following Krugman we call the two countries

N and S. At any moment there are a measure J of goods. Country N can produce

all of them (with unit efficiency), but S can produce only a subset J_S (also with unit

efficiency). In other words, goods in the set $J_N = J - J_S$ S can produce with only zero

efficiency. Competition is perfect.

With unit efficiency, a good produced in country i costs w_i. Since prices are

proportional to wage costs, spending on a typical N good relative to an S good is:

$$\frac{x_N}{x_S} = \left(\frac{w_N}{w_S}\right)^{1-\sigma}$$

where σ continues to represent the elasticity of substitution between products. If N

specializes in its exclusive goods then:

$$\frac{w_N L_N}{w_S L_S} = \frac{J_N}{J_S}\left(\frac{w_N}{w_S}\right)^{1-\sigma}.$$

The relative wage is thus:

$$\frac{w_N}{w_S} = \left(\frac{J_N/L_N}{J_S/L_S}\right)^{1/\sigma} \tag{3.22}$$

which needs to exceed 1 for N not to produce the S goods as well (in which case the

wage is the same in each place). The wage in N is larger the smaller its labor force

relative to S's and the larger the measure J_N of goods that are exclusive to it relative to the measure J_S that S can make as well.

Note the parallel between this expression for the relative wage with expression (3.21) yielded by the Ricardian model with a continuum of goods with efficiencies are realizations of random variables drawn from (3.20). Country i's range of goods J_i replaces the technology parameter T_i while the elasticity parameter σ replaces $\theta + 1$.

To this static formulation Krugman adds processes of innovation and imitation. Innovation is the development of new goods, which occurs according to the process:

$$\dot{J}(t) = \iota J(t) \tag{3.23}$$

where $J(t) = J_N(t) + J_S(t)$, the total measure of goods existing at date t. The parameter ι represents the rate of innovation and corresponds to the growth rate in the total measure of goods. Imitation occurs as knowledge of how to make exclusively northern goods diffuses to S. A given N good faces a hazard ϵ of diffusing to to S.[18] Hence:

$$\dot{J}_S(t) = \epsilon J_N(t). \tag{3.24}$$

Combining (3.23) and (3.24) implies that:

$$\dot{J}_N(t) = \iota J(t) - \epsilon J_N(t). \tag{3.25}$$

Krugman considers a steady-state in which $J_N(t)/J(t)$ is constant. Dividing (3.25) by

---

[18] Nelson and Phelps (1966) provide an earlier formulation of innovation and diffusion of this form.

$J_N(t)$ and insisting that $J_N(t)$ also grow at rate ι gives:

$$\frac{J_N(t)}{J(t)} = \frac{\iota}{\iota + \epsilon}.$$

Hence, in a steady state with a constant L_N/L_S, the steady-state wage ratio, in terms of the parameters of innovation and diffusion, is:

$$\frac{w_N}{w_S} = \frac{w_N}{w_S} = \left(\frac{\iota}{\epsilon}\right)^{1/\sigma} \left(\frac{L_N}{L_S}\right)^{-1/\sigma}.$$

An important implication of the model is that, in steady state, N and S each grow at the same rate in steady state. The price index is:

$$P(t) = \left[J_N(t)w_N^{1-\sigma} + J_S(t)w_S^{1-\sigma}\right]^{1/(1-\sigma)} = J(t)^{1/(1-\sigma)} \left[\frac{\iota w_N^{1-\sigma}}{\iota + \epsilon} + \frac{\epsilon w_S^{1-\sigma}}{\iota + \epsilon}\right]^{1/(1-\sigma)}, \quad (3.26)$$

which is common to both countries since there are no transport costs. Since the number of products grows at rate ι, the real wage rises in each location at rate $\iota/(\sigma-1)$. Country N is perpetually ahead of S, however. How far ahead depends on the rate of innovation relative to the rate of diffusion and relative labor forces. Hence the model captures a first-order features of Figures 14 and 15 in the previous chapter by providing a simple explanation for why countries can continue to grow at very similar rates but at very different levels of income.

3.2.2   Endogenous Innovation: Monopolistic Competition

Krugman's framework does not try to model the process of innovation itself, which became an active research area subsequently. Grossman and Helpman (1991a, 1991b)

endogenize the creation of new products in a dynamic version of monopolistic competition. We present their version as it applies to a closed economy.

As in Krugman (1979b) the measure of extant goods in period t is given at $J(t)$. From (3.26), the price index for a single economy is:

$$P(t) = J(t)^{1/(1-\sigma)} w(t)$$

Substituting into the expression for profit under monopolistic competition, (3.8) above, profit for a variety is then:

$$\pi(t) = \frac{X(t)}{\sigma J(t)}$$

where $X(t)$ is period t spending.

Posit a steady state in which $X(t)$ grows at rate g_X and $J(t)$ grows at rate g_J (both to be determined). Setting $w(t) = w$, an implication is that the inflation rate is $-g_J/(\sigma-1)$. Agents discount future profits at an exogenous rate ρ. Growth in spending X causes profit to grow over time while growth in the number of varieties causes profit per variety to shrink over time. The discounted value of profit at time t, taking into account future inflation, is thus:

$$V(t) = \frac{1}{\rho - g_X + [(\sigma - 2)/(\sigma - 1)]\, g_J}\frac{X(t)}{\sigma J(t)},$$

which corresponds to the value of developing a new good at time t.

In contrast with Krugman, innovation takes effort. One worker can innovate

at rate $\alpha(t)$, so that:

$$\dot{J}(t) = \alpha(t)r(t)L(t), \tag{3.27}$$

where $L(t)$ are the number of workers at date t and $r(t)$ the fraction engaged in research. The reward to research activity is $\alpha(t)V(t)$ while a worker earns the wage w making goods. Labor-market equilibrium thus requires that:

$$
\begin{aligned}
\alpha(t)V(t) &= w & r(t) \in [0,1] \\[2mm]
\alpha(t)V(t) &\leq w & r(t) = 0 \\[2mm]
\alpha(t)V(t) &\geq w & r(t) = 1.
\end{aligned}
\tag{3.28}
$$

Total spending is $X(t) = \overline{m}[1-r(t)]wL(t)$, the income of production workers augmented by the markup.

To close the model we need to specify how $L(t)$ and $\alpha(t)$ evolve. Two different approaches appear in the literature.

Grossman-Helpman treat L as fixed while efficiency in producing ideas for goods grows with "knowledge capital," proxied by the stock of goods already developed. Thus we can set $\alpha(t) = \alpha J(t)$. From (3.27):

$$g_J = \alpha r^* L$$

while $g_X = 0$. The condition for an interior labor-market equilibrium is now:

$$r^* = \frac{1}{\sigma - 1} - \frac{\rho}{\alpha L}$$

so that the growth rate in the number of products is:

$$g_J = \frac{\alpha L}{\sigma - 1} - \rho.$$

The growth rate increases in proportion to the population adjusted for research productivity.[19]

Jones (1995) provides an alternative formulation, treating research productivity α as fixed while letting L grow at an exogenous rate n. The ratio of goods $\varphi(t) = J(t)/L(t)$ evolves according to:

$$\dot{\varphi}(t) = \frac{\alpha r(t)L(t)}{L(t)} - \varphi(t)n.$$

In a steady state with $r(t)$ constant at r^*:

$$\varphi(t) = \varphi^* = \frac{\alpha r^*}{n}.$$

---

[19]This solution requires parameter values such that $r^* \in [0, 1]$. If the discount rate is too high, for example, there is no research or growth. The assiduous reader may note that our expression differs slightly from Grossman and Helpman's (1991b, p. 61). The reason is that they assume logarithmic preferences, while our assumption of fixed discount factor implies linear preferences. In their model the discount rate is equal to the exogenous rate of time preference ρ' plus the rate at which real consumption grows, $g_J/(\sigma - 1)$. To derive their expression from ours, replace our discount factor ρ with $\rho' + g/(\sigma - 1)$, to obtain:

$$g_J = \frac{\alpha L}{\sigma} - \frac{\sigma - 1}{\sigma}\rho'.$$

Translating our notation into theirs, this expression is their (3.28).

Since $g_X = g_J = n$, the value of an idea is:

$$V = \frac{n}{\rho(\sigma - 1) - n} \frac{(1 - r^*)w}{\alpha r^*}.$$

For profit to be finite we require that $\rho > n/(\sigma - 1)$.

Substituting into the condition for an interior labor-market equilibrium implies:

$$r^* = \frac{n}{\rho(\sigma - 1)}.$$

More research is done the higher the population growth rate relative to the discount factor and the elasticity of substitution.

B

3.2.3   Endogenous Growth: Quality Ladders

Grossman and Helpman (1991a, 1991b) provide an alternative model of innovation and growth with elements much closer to DFS's formulation of the Ricardian model rather than to monopolistic competition. In their model, the most efficient (or highest quality) technology for each good j is the consequence of a sequence of innovations, each one raising efficiency (or quality) over the previous state of the art by a factor $\lambda > 1$. Hence the most efficient technology for making good j if it has experienced $m(j)$ innovations is $z_0 \lambda^{m(j)}$, where z_0 is the efficiency level at date 0 (assumed constant across goods).[20]

As in DFS, preferences are Cobb-Douglas with equal share across the continuum of goods.

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[20] Aghion and Howitt (1992) provide a similar formulation.

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The state of the art technology for each good is proprietary. Potential producers of each good engage in Bertrand competition. The outcome is that only the most efficient technology is used for making each good. With Cobb-Douglas preferences, individual producers face unit elastic demand and charge the highest price that keeps the competition at bay. Hence a producer of a good j that has experienced $m(j)$ innovations charges a price $p(j) = w/(z_o\lambda^{m(j)-1})$, the unit cost using the previous state of the art. Since its own unit cost is $c(j) = w/(z_o\lambda^m)$ its profit is:

$$\pi(j) = [p(j) - c(j)]\frac{X(j)}{p(j)} = \frac{\lambda - 1}{\lambda}X$$

where, since preferences are Cobb-Douglas, spending $X(j)$ on good j is equal to total spending X. Note that profit is independent of the state of technology in the sector.[21]

Since spending goes either to profits or wages, $X(t) = \lambda wL(t)[1-r(t)]$. Again, $r(t)$ represents the share of workers engaged in research, so that $L(t)[1 - r(t)]$ workers produce output. We continue to treat the wage w as fixed over time.

With Cobb-Douglas preferences the price index $P(t)$ is:

$$P(t) = \exp\left[\int_0^1 \ln[p(j)]dj\right] = \frac{w\lambda}{z_0}\lambda^{-E[m(j)]}.$$

Innovations flow into the economy at rate ι (to be derived later) and are equally likely to apply to each good j. Since there are a unit continuum of goods, for

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[21]In contrast, in the monopolistic competition framework above, lower cost producers earn a higher profit. The reason is, with CES preferences with the elasticity of substitution greater than one, lower unit cost, which translates into a lower price, means higher sales.

70

any particular good j innovations arrive according to a Poisson process with arrival

rate ι.

With $r(t)L(t)$ workers engaged in research, ideas arrive at rate:

$$\iota(t) = \alpha r(t)L(t)$$

where again α is a parameter of research productivity. This model treats the labor

force L as constant.

Consider a steady state with r and ι also constant. The expected number of

innovations after a period of length t is ιt. Hence:

$$P(t) = w\lambda^{-\iota t} = w\exp[-(\iota\ln\lambda)t],$$

where, to simplify notation, we choose units so that $z_0 = \lambda$. The inflation rate in the

economy is thus $-\iota\ln\lambda$. Since w and L are fixed, the real growth rate is $\iota\ln\lambda$.

The term ι is also the hazard with which the current state of the art for

producing a good j is surpassed, at which point the owner of the surpassed invention

no longer earns a profit. With a discount rate of ρ, taking into account inflation and

the hazard of obsolescence, the value of a state of the art idea at time t is:

$$V = \frac{(\lambda - 1)wL(1 - r)}{\rho + \iota - \iota\ln\lambda} = \frac{(\lambda - 1)wL(1 - r)}{\rho + \alpha rL(\lambda - \ln\lambda).}.$$

Note that a higher rate of innovation ι has a positive effect on the value of an idea by

creating economic growth, which causes the real value of profit to rise over time. But

it has a negative effect by increasing the hazard of obsolescence. For small inventive steps (λ near 1), the negative obsolescence effect dominates.

Again, labor market equilibrium requires (3.28) above. At an interior solution the steady-state research share is:

$$r^* = \frac{\lambda - 1}{\Lambda} - \frac{\rho}{\alpha L \Lambda}$$

where $\Lambda = \lambda - \ln \lambda$. The implied rate of innovation is:

$$\iota = \frac{\lambda - 1}{\Lambda} \alpha L - \frac{\rho}{\Lambda}.$$

Again, growth increases with the labor force adjusted for research productivity.[22]

Grossman and Helpman (1991b) go on to develop two-country extensions of these dynamic models with technology diffusion and trade, examining the impact of various policies.

A
3.3 Further Reading

We've chosen to highlight a set of key results on technology in the global economy that set the stage for our own analysis. We have not provided an extensive survey of the

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[22] Again, conditions on parameters need to be imposed to guarantee that $r^* \in [0, 1]$. To obtain Grossman and Helpman's (1991b, p. 96) result with logarithmic preferences, replace our fixed discount factor ρ with their pure rate of time preference ρ' plus the growth in real income $\iota \ln \lambda$. Translating our notation into theirs delivers their expression (4.18).

literatures. The reader eager to expand her knowledge is fortunate to have a number of excellent surveys to turn to.

Grossman and Helpman (1995) provide an analytic overview of the general literature on trade and technology, as it stood in the mid 1990's. For a detailed discussion of the theoretical literature on monopolistic competition and its relationship to the econometrics of the gravity equation, we recommend Chapter 5 of Feenstra (2004). For a comprehensive review of efforts to measure bilateral trade costs we refer the reader to Anderson and van Wincoop (2004).

Various aspects of growth in a multi-country context are surveyed by Klenow and Rodriguez-Clare (2005) and Benhabib and Spiegel (2005). Keller (2004) provides a comprehensive survey of work on the theory and empirics of the diffusion of technologies across countries.

A

3.4   Assessment

How successfully do these various models explain the features of the data described in the previous chapter? The literatures on gravity and monopolistic competition in open economies provide an explanation for the role of size and distance in bilateral trade flows. But when it comes to dissecting these flows into their producer and commodity components, these models come up short. The Armington approach, with no individual commodities, makes no prediction at all. Monopolistic competition, on the other hand,

makes very stark predictions both about exports and imports. But they are very much at variance with what evidence we have. The model predicts that variation in imports across countries is totally at the intensive margin, while the evidence is that imports vary across destinations mostly because of the number of firms selling. The basic version of trade with monopolistic competition makes the opposite prediction for exporters, that differences in export amounts across countries represent differences in the number of units selling. The evidence from detailed product categories suggests that this is 60 percent true, so that 40 per cent is accounted for by the amount of given products sold. That is, big countries sell more of the same good almost to the extent to which they sell a greater variety of goods.

The models of innovation and diffusion can explain why, over the long run, a common underlying process can generate very similar growth rates in different countries, while relative differences in income remain, very much the pattern we see in the data. But the models provide a parameterization of this phenomenon only in a two-country setting. As the data indicate, many countries both innovate and make use of the inventions of others.

In order to make their points as cleanly as possible, the models we have just discussed treat goods, producers, and inventions as identical or symmetric. A feature of the producer-level data, however, is the vast heterogeneity of producers in terms of size and where they sell. Data on cross-country patenting suggest that inventions also

vary enormously in their importance and the geographic breadth of their applicability.

The next section of the book provides an framework for analyzing trade and growth in a multicountry world. It uses many of the elements of the models we just described in order to explain bilateral trade patterns and the phenomenon of parallel growth that we observe in the data. Additional features allow us to come to terms with producer-level heterogeneity and the complex patterns of producer level participation in trade, and to understand patterns of innovation and diffusion in a multipolar world.

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