Innovation, Diffusion, and Trade

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Abstract

We consider the roles of trade and technology diffusion on the incentives for innovation. To some extent, diffusion substitutes for trade. If trade costs are low relative to differences in comparative advantage in doing research, rapid diffusion implies specialization in research activity. Country size has an ambiguous effect on specialization in research.
1 Introduction

Research indicators reveal strong and consistent patterns of specialization in innovation. Most research is done in the United States, Japan, and Germany, while only a small group of countries allocate much in the way of resources to inventive activity. Most countries, even within the OECD, do very little research. Moreover, with the exception of the emergence of Finland and Ireland as research centers, patterns of research specialization have remained very stable over time. The major research centers now are largely the same ones as a century ago.¹

What characteristics of a country determine specialization in research, and how does openness affect the incentive to innovate? This question has been posed in a number of contexts in which openness has meant different things. It could refer to the absence of trade barriers, but also to the absence of barriers to the diffusion of ideas themselves. While trade allows consumers in another country to benefit from an innovation by importing a good that embodies the idea, technology diffusion allows them to benefit through local production that uses the idea. Expanding one conduit or the other may have very different implications for the incentive to innovate in either location. A related question is the role of country characteristics in determining international patterns of specialization in innovative activity under differing degrees of openness to trade and knowledge. Do large countries, for example, naturally do more research because inventions have swifter access to a large internal market?

The literature on international technology diffusion is large. Keller (2004) provides a comprehensive survey. A number of papers have looked at the effect of one type of openness taking the degree of openness of the other type as given. Examples are Helpman (1993), Eaton, Gutierrez, and Kortum (1998), Eaton and Kortum (1999), and Eaton and Kortum (2001). Helpman (1993), for example, finds in a model with an innovating and imitating country, with costless trade and the absence of intellectual property protection in the imitating country, ¹Eaton, Gutierrez, and Kortum (1998), Eaton and Kortum (1999), and Eaton, Kortum, and Lerner (2003) provide alternative views of the research and patenting data.
faster diffusion can spur innovation by reducing the wage, and hence the cost of innovation, in the innovating country. In a model with no diffusion, Eaton and Kortum (2001) find that the degree of openness to trade has no effect on innovative activity: While trade increases the size of the market that a successful innovator can hope to capture, it also means that an innovator faces a higher hurdle in terms of competition from abroad. In their model, unlike Helpman, all countries engage in innovative activity.

To explore these issues further we develop a two-region model, like Helpman’s, of innovation and diffusion. In contrast to much of the literature, innovation can in principle take place in either region, although research productivities may differ. Ideas can diffuse the locations, but with a lag. We allow for an arbitrary level of trade barriers, with costless trade a special case. We then explore the incentives to innovate under different assumptions about the speed of diffusion and about the magnitude of trade barriers.

While our model could be extended to analyze the implications of imperfect protection of intellectual property, that is not our purpose here. To isolate the effects of geographical barriers to the movement of goods and ideas, we make the simplest assumption for our purposes, that innovators can appropriate the entire returns to their innovation at home and abroad.\(^2\)

We proceed as follows:

Section 2 develops a static two-country model of technology, production, and trade along the lines of the Ricardian model developed in Eaton and Kortum (2002). In their many-country model, the distribution of efficiencies is treated as independent from country to country. Such an outcome is consistent, for example, with a world in which each country relies on its own innovations for production, or one in which innovations applying to a particular good in one country apply to some different one where they diffuse. Here we consider the more natural, but much more complicated, case in which an innovation, when it diffuses, applies to the same good. This extension forces us to distinguish between innovations that are in the exclusive

\(^2\)The role of intellectual property protection was a main concern of Helpman (1993). The issue has been revisited recently by Ganica (2003) and by Dinopoulos and Segerstrom (2004).
domain of the innovating country, and those that have diffused to a common pool that the 
other country can access. Because of the many different situations that can arise, we limit 
ourselves to a two-country case. Even here we need to distinguish among situations in which: 
(i) one country uses only those ideas that are exclusive to it, leaving the ideas that have 
flowed into the common pool to the other country, (ii) both countries use ideas that have 
flowed into the common pool, with one country exporting goods produced using those ideas 
to the other, and (iii) both countries use ideas that have flowed into the common pool, with 
no trade in goods produced using those ideas. The first case replicates the situation in Eaton 
and Kortum (2002), since the efficiencies that each country actually uses are drawn from 
independent distributions. Diffusion has no impact on the extent of trade. In the second 
two cases, diffusion substitutes for trade as at least a range of the goods produced with the 
common technology are nontraded. Nontradedness arises not because transport costs for these 
goods are higher, but because similarities in efficiency have eliminated any scope for exploiting 
comparative advantage.

Section 3 introduces simple dynamics into the analysis. Each country innovates at an 
exogenous rate, and ideas diffuse from one to another at exogenous rates. The processes of 
innovation and diffusion generate a world steady-state growth rate in which the two countries, 
depending on their abilities to innovate and to absorb ideas from abroad (except by coinci-
dence) have different relative income levels. The framework can deliver ”product cycles,” as in 
Krugman (1979), in which the innovator initially exports the good using the technology it has 
developed, but later imports it once the technology has diffused abroad. In our model other 
outcomes are possible, however. If the innovation is sufficiently small, before diffusion, the 
other country may continue to produce the good on its own using inferior technology rather 
than import the good from the innovator. In fact, its own technology could even be superior, 
so that the innovation is never useful outside the country of innovation.

Section 4 endogenizes the innovation process. It first solves for the value of ideas in each
country, and then calculates the incentive to innovate. The trade-off between the returns to innovation and to production determine the extent of innovation in each country. Here we consider the role of openness in the form of (i) lower trade barriers and (ii) faster diffusion on innovative activity on innovation in each country. A special case is no diffusion, returning is to Eaton and Kortum (2001). Each country allocates the same share of resources to inventive activity regardless of its size or relative research productivity. Turning to the other extreme of immediate diffusion, we find that the same result emerges if the trade barrier exceeds the ratio of research productivities. But with if the trade barrier is lower than the ratio of research productivities, the more efficient researcher specializes in innovation along Ricardian lines.

Section 5 offers some concluding remarks.

2 A Model of Technology, Production, and Trade

Our production structure is Ricardian. Following, for example, Dornbusch, Fischer, and Samuelson (1977, henceforth DFS) we consider a world with a unit continuum of goods, which we label by $j \in [0,1]$. There are two countries, which we label $N$ (for North) and $S$ (for South). Each country has a set of available technologies for making each of the goods on the continuum. Some technologies, denoted $N$, are available only to the North while another set, $S$, are available exclusively to the South. A third set $C$ are commonly available. A technology is the ability to produce $z_i(j)$ units of good $j$ with one worker, where, depending on which type of technology we are talking about, $i = N, S, C$.

We treat the $z_i(j)$’s as realizations of random variables $Z_i$ drawn independently for each $j$ from the Frechet distributions:

$$F_i(z) = \Pr[Z_i \leq z] = \exp[-T_i z^{-\theta}].$$

which are independent across $i = N, S, C$. In this static context the $T_i$’s reflect the differences in average efficiencies across the three sets of technologies. (We consider how these distributions arise from a dynamic process of innovation and diffusion in Sections 3 and 4.)
The best technologies available in country $i$ are:

$$Z_i^* = \max[Z_i, Z_C] \ i = N, S.$$ 

Thus the variable $Z_i^*$ has distribution:

$$F_i^*(z) = \Pr[Z_i^* \leq z] = \exp[-T_i^* z^{-0}]$$

where $T_i^* = T_i + T_C$.

Eaton and Kortum (2002) consider a case in which there is no common technology, so that $T_C = 0$. An implication is that the distributions of efficiencies available to each country are independent. Here there is independence across the exclusive technologies, but the common technologies induce a positive correlation between $Z_N^*$ and $Z_S^*$. Because of this correlation we will find it easier to work with the three independent technologies $Z_N, Z_S, \text{and } Z_C$. 

As is standard in a Ricardian setting, workers are identical and mobile across activities within a country, but cannot change countries. The wage is $w_N$ in the North and $w_S$ in the South. We take the wage in the South to be the numeraire, although we sometimes leave $w_S$ in formulas for clarity. The labor market clearing conditions establish the relative wage. Without loss of generality we will impose restrictions on exogenous variables so that in equilibrium $w_N \geq w_S$.

As in DFS, demand is Cobb-Douglas. Hence expenditure in country $i$ on good $j$ is:

$$X_i(j) = Y_i.$$ 

where $Y_i$ is total expenditure.$^3$

Goods can be transported between the countries, but in order to deliver one unit in the destination $d \geq 1$ units need to be shipped from the source (the standard “iceberg” assumption).

Unfortunately, even in low dimensional Ricardian problems, taxonomies are inevitable. There

$^3$Below we consider the case in which technologies and the labor forces evolve over time. Since in this section we solve the static equilibrium given these magnitudes, We omit time subscripts for now.
are three possible cases to consider: (1) If, in equilibrium, $w_N > w_S$ then the commonly available technology will be used only in the South; the North will use only those technologies that it has unique access to. (2) If in equilibrium $w_N = w_S$ then the commonly available technology may be used in both countries, but goods produced using this technology are exported only by the South. (3) If in equilibrium $w_N < w_S$ then each country will use the commonly available technology. Goods produced using this technology are not traded since it will be more expensive to import the good than to make it oneself.

2.1 Cost Distributions

To mitigate the proliferation of special cases, we introduce the notation $w_{ni}$ for the effective wage (inclusive of transport cost) for goods sold in country $n$ produced using technology $i$. Here $n = N, S$ and $i = N, S, C$. For example, $w_{NC} = \min \{w_Sd, w_N\}$ is the wage paid to labor producing goods using the common technology and sold in the North (including transport cost should the South be the sole user). Hence $w_{NC}/z_C(j)$ is the cost of selling good $j$ in the North if it is produced using the common technology. In the first case above $w_{NC} = w_Sd < w_N$; in the second $w_{NC} = w_N = w_Sd$; in the third, $w_{NC} = w_N < w_Sd$.

The least cost means of obtaining good $j$ in the North is thus:

$$c_N(j) = \min \{w_N/z_N(j), w_{NC}/z_C(j), w_Sd/z_S(j)\}$$

while in the South it is:

$$c_S(j) = \min \{w_Nd/z_N(j), w_S/z_C(j), w_Sd/z_S(j)\}$$

since the South always buys goods using the common technology from itself.

For $i = N, S$, the lowest cost $c_i(j)$ is the realization of a random variable $C_i$ whose distribution is determined by the distribution of the underlying technologies $Z_i$. We denote the cost
distribution in $i$ by $H_i(c) = \Pr[C_i \leq c]$. The cost distribution in the North is:

$$
H_N(c) = 1 - \Pr[Z_N \leq w_N/c] \Pr[Z_S \leq w_Sd/c] \Pr[Z_C \leq w_{NC}/c]
$$

$$
= 1 - F_N(w_N/c)F_S(w_Sd/c)F_C(w_{NC}/c)
$$

$$
= 1 - \exp\left[-\Phi_N c^\theta\right]
$$

where $\Phi_N = T_N w_N^{-\theta} + T_S (w_Sd)^{-\theta} + T_C w_{NC}^{-\theta}$. Similarly, for the South:

$$
H_S(c) = 1 - \Pr[Z_N \leq w_Nd/c] \Pr[Z_S \leq w_S/c] \Pr[Z_C \leq w_S/c]
$$

$$
= 1 - \exp\left[-\Phi_S c^\theta\right]
$$

where $\Phi_S = T_N (w_Nd)^{-\theta} + T_S w_S^{-\theta} + T_C w_{S}^{-\theta} = T_N (w_Nd)^{-\theta} + T_S w_S^{-\theta}$.

As shown in EK (2002), the probability that country $n$ will find technology $i$ the lowest cost source for some good is:

$$
\pi_{ni} = \frac{T_i w_{ni}^{-\theta}}{\Phi_n}
$$

where $n = N, S$ and $i = N, S, C$.

### 2.2 Trade Patterns and Wages

Under perfect competition or under Bertrand competition, $\pi_{NN}$ is the fraction of the North’s expenditure devoted to goods produced with exclusively Northern technology while $\pi_{SN}$ is the fraction the South spends on goods produced with the exclusively Northern technology. If $Y_n$ is total spending by country $n$, spending on labor producing goods produced using Northern technology is:

$$
w_N L_N^E = \beta (\pi_{NN} Y_N + \pi_{NS} Y_S)
$$

Here $\beta$ is the labor share in production and $L_N^E$ are the measure of Northern workers using exclusively Northern technology.

We need to distinguish the three kinds of equilibria:
2.2.1 Case 1: The North uses only its Exclusive Technology

In this case, \( L_N^E = L_N^P \), where \( L_N^P \) are all Northern workers engaged in production. Since only the South uses the commonly available technology, \( w_{NC} = w_{SD} \). The solution needs to satisfy the condition that \( w_N > dw_S \) in order for the North not to use the commonly accessible technology.

Under perfect competition, all \( L_i \) workers in each are engaged in production and labor is the only source of income, so that \( L_i^P = L_i, \beta = 1 \) and \( Y_i = w_iL_i \). In this case the expression above becomes:

\[
\frac{w_N}{w_S} = \left[ \frac{T_N/L_N}{T_S/L_S} \left( \frac{w_N^{-\theta} + T_S^*(w_{SD})^{-\theta}}{T_N(w_{ND})^{-\theta} + T_S^*w_S^{-\theta}} \right)^{1/(1+\theta)} \right]. 
\]

While the equation does not admit an analytic solution it is easy to solve numerically. Later on we will introduce a research activity and imperfect competition, so that there are other sources of income and not all spending goes to labor.

Since the North does not use any of the commonly produced technology, all goods produced are equally tradable regardless of which technology they employ. The fact that the North has access to the common technology is irrelevant since it doesn’t use it. The outcome is isomorphic to one in which the North only knows the technologies that are exclusive to it, while the common technologies are exclusive to the South, as in EK (2002).

2.2.2 Case 2: The North and the South both use the Common Technology, with Trade in Some Goods Produced using that Technology

In this case \( w_{NC} = w_N = w_{SD} \). Under perfect competition the demand for workers using the exclusive Northern technology is:

\[
\frac{L_N^E}{L_N} = \frac{T_N}{T_W} + \frac{T_Nd^{-2\theta}}{T_Nd^{-2\theta} + T_S^*dL_N}.
\]

where \( T_W = T_N + T_C + T_S \), a measure of world technology. For this case to emerge parameter values must be such that \( L_N^E/L_N \) does not exceed one. Otherwise we are in case 1 above. We
also need that the demand for workers using the South’s exclusive technology $L_S^S$ not exceed the supply of Southern workers. This condition requires that the ratio

$$\frac{L_S^S}{L_S} = \frac{T_S}{T_W} \frac{dL_N}{L_S} + \frac{T_S}{T_N d^{-2\theta} + T_S^*}$$

not exceed one. Otherwise we are in case 3 below.

In this case the range of goods produced using the common technology in the North are not traded. Hence, unlike case 1, technology diffusion results in less trade than otherwise. Diffusion substitutes for trade.

### 2.2.3 Case 3: Goods Produced with the Common Technology are not Traded

In this case $w_{NC} = w_N < w_S d$. Labor market equilibrium requires a wage $w_N$ that solves:

$$w_N L_N^P = \beta [\pi_{NN} + \pi_{NC}] Y_N + \pi_{NS} Y_S$$

which, under perfect competition, becomes:

$$w_N = \left[ \left( \frac{T_N/L_N}{T_S/L_S} \right) \frac{T_N w_N^{-\theta} + T_S (w_S d)^{-\theta}}{T_N (w_N d)^{-\theta} + T_S^* w_S^{-\theta}} \right]^{1/(1+\theta)}$$

Again, there is no analytic solution but solving for the wage numerically is straightforward.

All goods produced using the common technology are not traded. In this case technology diffusion reduces the scope for trade even further.

### 2.3 Trade and Prices

What is the relationship between technology, wages, and prices in each of these cases? In cases 1 and 2 the wage in the North is higher than that in the South by a factor of at least $d$ while the prices of goods produced using the common technology are higher by a factor of exactly $d$. In case 2, some of these goods are produced in both countries using the same technology. In contrast, goods produced with the Northern technology are more expensive in the South by a factor $d$. In this case the model delivers the (Balassa-Samuelson) implication that nontraded goods are cheaper in the South.
3 Simple Technology Dynamics

We have so far considered the static equilibrium in which labor forces and levels of technology are given. Over time, however, we can envisage processes of innovation and diffusion governing the evolution of $T_{Nt}$, $T_{St}$, and $T_{Ct}$ (introducing a time subscript). We first follow the specification in Krugman (1979), for example, which allows us to stick with perfect competition: Each country innovates at an exogenous rate that is proportional to its current knowledge, and that ideas flow from the exclusive to the common pool at rates that are proportional to the stocks of exclusive ideas. We introduce four parameters, $\iota_N$, the rate at which the North innovates, $\iota_S$, the rate at which the South innovates, $\epsilon_N$, the rate at which the South learns about exclusively Northern ideas, and $\epsilon_S$, the rate at which the North learns about exclusively Southern ideas: Thus $T_{Nt}$, $T_{St}$, and $T_{Ct}$ evolve over time according to:

$$\frac{dT_{Nt}}{dt} = (\iota_N - \epsilon_N)T_{Nt} + \iota_NT_{Ct}$$

$$\frac{dT_{St}}{dt} = (\iota_S - \epsilon_S)T_{St} + \iota_ST_{Ct}$$

$$\frac{dT_{Ct}}{dt} = \epsilon_NT_{Nt} + \epsilon_ST_{St}.$$ 

While the analytic solution to this dynamic system is complex, it is straightforward to show that as long as the innovation and diffusion parameters are strictly positive and the initial value of at least one $T_i$ is positive, the system evolves to a steady state in which all three types of knowledge grow at the same rate.

In general, the resulting growth rate is the solution to an unpleasant cubic equation. It can be shown, however, that the steady-state growth rate is increasing in both the innovation and diffusion parameters (see, e.g., Eaton and Kortum, 1999). In the special case of symmetry, $\iota_N = \iota_S = \iota$, and $\epsilon_N = \epsilon_S = \epsilon$, the steady-state growth rate is merely quadratic:

$$g = \frac{\iota - \epsilon + \sqrt{(\iota - \epsilon)^2 + 8\iota\epsilon}}{2},$$

strictly increasing in $\iota$ and $\epsilon$. A world with more innovation but also more diffusion grows
faster. Krugman (1979) considers a special case in which only the North innovates, so that $i_S = 0$ and the growth rate is just $i_N$ while $T_S = 0$.

4 Endogenizing Innovation

We now amend the model to endogenize innovation. We continue to assume that exclusive ideas flow into common knowledge at a common exogenous rate $\epsilon$.

As in Kortum (1997), we model innovation as the production of ideas. An idea is a way to produce a good $j$ with output per worker $q$. We assume that an idea is equally likely to apply to any good in the unit interval, and that $q$ is the realization of a random variable $Q$ drawn from the Pareto distribution:

$$F(q) = \Pr[Q \leq q] = 1 - q^{-\theta}. \quad (5)$$

Only an idea that lowers the cost of serving a market will be used.

Initially, ideas innovated in country $n$ will only be usable for production there. Hence for an idea from country $n$ to lower the cost of serving the home market $q$ must satisfy:

$$w_n/q \leq c_n(j) = \min[w_{nN}/z_N(j), w_{nS}/z_S(j), w_{nC}/z_C(j)],$$

where $z_N(j)$, $z_S(j)$, and $z_C(j)$ represent the states of the art in the exclusively Northern, exclusively Southern, and commonly available technologies, respectively. (Recall, $w_{ni}$ is the transport cost-inclusive wage applying to technology $i$ in market $n$.) To lower the cost of serving the foreign market $m \neq n$ it must satisfy:

$$w_n d/q \leq c_m(j) = \min[w_{mN}/z_N(j), w_{mS}/z_S(j), w_{mC}/z_C(j)]$$

This second criterion is more stringent. Hence a small innovation may be used only for production for the home market (at least before diffusion), while a larger one will be used to produce both for the home and for export markets.
Given the current local cost \(c_n(j)\) of good \(j\), the probability that a local innovation will lower cost is:

\[
\Pr\left[ \frac{w_n}{Q} \leq c_n(j) \right] = \Pr\left[ Q \geq \frac{w_n}{c_n(j)} \right] = \left[ \frac{w_n}{c_n(j)} \right]^{-\theta}
\]

while given the cost in the foreign market \(m\), \(c_m(j)\), the probability that it lowers cost abroad is:

\[
\Pr\left[ \frac{w_n}{Q} \leq c_m(j) \right] = \Pr\left[ Q \geq \frac{w_n}{c_m(j)} \right] = \left[ \frac{w_n}{c_m(j)} \right]^{-\theta}
\]

Again, since costs can never differ by more than a factor \(d\), and can differ by less, the criterion for exporting is tougher.

We need to introduce an incentive for innovation. We follow the quality ladders framework (Grossman and Helpman, 1991, Aghion and Howitt, 1992) and posit that the owner of an innovation has the ability to use it to produce and sell a product at the highest price that keeps the competition at bay. Thus an innovator who can produce good \(j\) at unit cost \(c^{(1)}(j)\) can set a price \(c^{(2)}(j)\), the unit cost of the second lowest cost producer.

### 4.1 The Distribution of the Mark Up

Consider the markup \(M_{nit} = C_{nt}/(w_{ni}/Q)\) of a good produced with an idea in technology class \(i = N, S, C\) in market \(n = N, S\). (Here capital letters denote random variables.) The efficiency \(Q\) is drawn from the distribution 5) while the alternative cost \(C_{nt}\) is drawn from the cost distribution:

\[
H_n(c) = 1 - \exp\left[-\Phi_n c^\theta \right].
\]
from (2) and (2). The probability that the markup exceeds some value $m$ is:

$$b_{nit}(m) = \Pr[M_{nit} \geq m] = \Pr[C_{nt} \geq w_{ni}m/Q]$$

$$= \int_1^\infty \Pr[C_{nt} \geq mw_{ni}/Q|Q = q]q^{-\theta-1}dq$$

$$= \int_1^\infty \exp \left[-\Phi_{nt}(mw_{ni})^\theta q^{-\theta}\right]q^{-\theta-1}dq$$

$$= \frac{m^{-\theta}}{\Phi_{nt}w_{ni}^\theta}$$

(Since we consider a steady state in which $w_N$ is constant, we do not index it by $t$.) For the good to be sold, of course, requires $M \geq 1$, which occurs with probability:

$$b_{nit}(1) = \frac{1}{\Phi_{nt}w_{ni}^\theta}$$

which we can rewrite:

$$b_{nit}(1) = \frac{\pi_{ni}}{T_{it}}$$

An idea will be used if and only if $m \geq 1$. The distribution of the markup conditional on $m \geq 1$ is just:

$$G(m) = \frac{b_{nit}(1) - b_{nit}(m)}{b_{nit}(1)} = 1 - m^{-\theta} \quad n = N, S; \quad i = N, S, C.$$  
the simple Pareto distribution with parameter $\theta$.

Integrating across the markup distribution $G(m)$, the expected flow of profit from an idea conditional on selling in country $n$ is:

$$Y_{nt} \int_1^\infty (1 - m^{-1})dG(m) = \frac{Y_{nt}}{1 + \theta}; \quad n = N, S$$

which is also the total profit generated in country $n$. The fraction earned by using technology of type $i = N, S, C$ is $\pi_{ni}Y_{nt}/(1 + \theta)$. Taking into account the probability that an idea will be useful in that country, the expected profit in market $n$ of an idea from technology $i$ at time $s$ is:

$$\Pi_{nis} = \frac{1}{1 + \theta} \frac{\pi_{ni}Y_{ns}}{T_{is}}$$
where $P_{ns}$ is the price level in country $n$ at time $s$. As shown in Eaton and Kortum (2001), $P_{nt} = \gamma \Phi_{nt}^{-\theta}$. As time passes, $Y_{nt}$ and $T_{it}$ both grow. The first causes expected profit to from an idea rise over time as the size of the market grows. The second causes expected profit from the idea to fall over time through the hazard of losing the market to a cheaper source of production. Since we use the Southern wage as numeraire, growth in the $T_i$’s causes $\Phi_n$ to fall over time.

With these expressions in hand we are now armed to calculate the value of ideas.

### 4.2 The Value of Ideas

Taking into account the potential for diffusion, the expected value of an idea developed in country $i$ over its lifetime is:

$$
V_{it} = \frac{P_{it}}{1 + \theta} \int_0^\infty \frac{e^{-\rho(s-t)}}{P_{is}} \left[ e^{-\epsilon(s-t)} \left( \frac{\pi_{Ni}Y_{Ns} + \pi_{Si}Y_{St}}{T_{is}} \right) + \left[ 1 - e^{-\epsilon(s-t)} \right] \left( \frac{\pi_{NC}Y_{Ns} + \pi_{SC}Y_{St}}{T_{Cs}} \right) \right] ds.
$$

The expression $\pi_{Ni}Y_{Ni} + \pi_{Si}Y_{Si}$ corresponds to total spending on goods using technology $i$.

We assume that both labor force grows at rate $n$, with $L_N = \lambda L_S$. In steady state wages and the $\pi_{ni}$ are constant, while the $T$’s grow at rate $g$. Profit is a constant share of income, which also grows at rate $n$.

We can write the income of country $i$ $Y_i$ n terms of its output $Q_i$ as:

$$
Y_i = (1 - \frac{\omega_{ji}}{1 + \theta})Q_i + \frac{\omega_{ij}}{1 + \theta}Q_j
$$

where $\omega_{ij}$ is the share of the technology used in country $i$ owned by inventors from country $j$, $j \neq i$. Since labor income from production in country $i$ is $w_iL_{it}^P$, where $L_{it}^P$ denotes production workers, we can relate output to labor income there as follows:

$$
Q_{it} = w_iL_{it}^P + \frac{Q_{it}}{1 + \theta} = \frac{1 + \theta}{\theta}w_iL_{it}^P.
$$
Hence:

\[ Y_i = \frac{1}{\theta} \left[ (1 + \theta - \omega_{ij})w_i L_i (1 - r_i) + \omega_{ji} w_j L_j (1 - r_j) \right] \]  

(7)

We use these expressions for income in deriving the value of ideas in the various cases we consider below.

4.3 The Rate of Innovation

We define the ratio of technology exclusive to country \( i \) to country \( i \) workers at time \( t \) as \( t_{it} = T_{it}/L_{it} \). It evolves according to:

\[
\frac{\dot{t}_{it}}{t_{it}} = \frac{\dot{T}_{it} - L_{it}}{T_{it} - L_{it}} = \frac{\alpha_i r_i L_{it}}{T_{it}} - (n + \epsilon).
\]

where \( r_i \) is the share of the labor force in country \( i \) engaged in research. In steady state both \( r_i \) and \( t_i \) are constant, so that \( t_i = \alpha_i r_i/(n+\epsilon) \) and \( g = n \). Finally, defining \( t_{Ct} = T_{Ct}/(T_{Nt} + T_{St}) \), the ratio of common to exclusive technologies, \( t_C \) evolves according to:

\[
\frac{\dot{t}_{Ct}}{t_{Ct}} = \epsilon - t_{Ct} \frac{T_{Nt} + T_{St}}{T_{Nt} + T_{St}} = \epsilon - t_{Ct} n.
\]

Thus in steady state \( t_C \) is constant and equals \( \epsilon/n \).

In steady state \( g = n \), our expression for the value of ideas (6) becomes:

\[
V_{it} = \frac{1}{\rho + \epsilon - n/\theta} \left[ \frac{\pi_{Nt} Y_{Nt} + \pi_{St} Y_{St}}{(1 + \theta)T_{it}} + \left( \frac{1}{\rho - n/\theta} - \frac{1}{\rho + \epsilon - n/\theta} \right) \frac{\pi_{NC} Y_{Nt} + \pi_{SC} Y_{St}}{(1 + \theta)T_{Ct}} \right].
\]

(8)

where income is given by (7) above. In addition, the steady state values of the foreign shares of royalty in GDP are:

\[
\omega_{ji} = \frac{T_i}{T_i + T_j} \frac{T_C}{T_t + T_j + T_C} \frac{\epsilon R_{ij}}{(n + \epsilon) + \epsilon R_{ij}}.
\]
where $R_{NS} = \alpha_N r_N / \alpha_S r_S$ reflects the scale of research effort in the North relative to the South, and $R_{SN} = 1 / R_{NS}$.

In an equilibrium in which workers in a country engage simultaneously in production and innovation, the return to each should be equal. More generally, the conditions for labor market equilibrium are that:

$$\alpha_i V_{it} = w_{it} \quad r_{it} \in [0, 1]$$

$$\alpha_i V_{it} \leq w_{it} \quad r_{it} = 0$$

$$\alpha_i V_{it} \geq w_{it} \quad r_{it} = 1.$$  

for $i = N, S$.

A steady state equilibrium is a solution for $r_N, r_S$, and $w_N/w_S$ consistent with (9) and product market clearing as derived in Section 2.

4.4 Steady State Research and Growth

Because of the taxonomy of situations that can arise, we avoid trying to provide a general analytic solution. But under each of four particular assumptions about diffusion and trade barriers the model yields insight into the effect of globalization on research and income: (i) no diffusion ($\epsilon = 0$), (ii) instantaneous diffusion ($\epsilon \rightarrow \infty$), (iii) no trade costs. ($d = 1$) and (iv) infinite trade costs ($d \rightarrow \infty$). As these four cases circumnavigate the full range of possibilities, they provide insight into intermediate cases:

4.4.1 No Diffusion

Setting $\epsilon = 0$ gives us EK (2001). Each country has to use its own ideas for production. Hence, spending on goods produced with ideas exclusive to country $i$ corresponds with the total production of country $i$, $Q_i = \pi_{Ni} Y_{Nt} + \pi_{Si} Y_{St}$. Substituting this expression into value of ideas above and solving for labor market equilibrium in each country we obtain:

$$r = \frac{n}{\rho \theta_i}.$$
In steady state all countries do the same amount of research regardless of their size or their research productivity. We now explore the extent to which international technology diffusion upsets this stark result.

4.4.2 Instantaneous Diffusion

Consider the opposite case in which diffusion is infinite ($\epsilon \to \infty$). Now all ideas are common, so that $\pi_nC = \pi_nC = 1, n = N, S$. Since all ideas diffuse immediately they have the same value regardless of their origin. We can write the value of an idea (regardless of source) as:

$$V = \frac{1}{\rho^\theta - n} \frac{w_N L_N (1 - r_N) + w_S L_S (1 - r_S)}{T_C}.$$

Define $\tilde{t} = T_C / L_N$. Differentiating with respect to time reveals that in steady state:

$$\tilde{t} = \frac{\alpha_N r_N \lambda + \alpha_S r_S}{n \lambda}.$$

Since each country has access to the same technology, as long as the North continues to engage in production, $w_N \leq w_S d$. (Remember that $w_N \geq w_S$ throughout our analysis.) What happens depends on the relative size of $\alpha_N / \alpha_S$ and $d$.

1. Say that $\alpha_N / \alpha_S \geq d$. Then as long as the North continues to produce, $w_N = w_S d$ and the North does all the research ($r_S = 0$). The fraction of research done in the North is:

$$r_N = \frac{n}{\rho^\theta} (1 + 1/d \lambda) \quad (10)$$

which cannot exceed one if the North is still producing. Note that, compared with the case of no diffusion, the amount of research is higher in the North in proportion to the relative size of the South ($1/\lambda$), discounted by the trade barrier ($1/d$). Because of this discounting, the number of people engaged in research in the world is smaller than with no diffusion, but since $\alpha_N / \alpha_S \geq d$, effective research is higher since Northern research workers are more productive. This condition ensures that if the North is both producing and doing research, the South does not find research worthwhile. In this equilibrium the
South runs a trade surplus with the North to pay for the ideas it uses in production. The location of production of any particular good is indeterminate.  

2. Say instead that $\alpha_N/\alpha_S \leq d$. In this case both countries will engage in research, and the relative wage will reflect research productivity, i.e., $w_N/w_S = \alpha_N/\alpha_S \leq d$. Since the trade barrier exceeds the wage difference, each country will produce its own goods. Since there is no goods trade, royalty payments must balance, which requires that $r = r_N = r_S$. For each country to find research worthwhile requires that:

$$r = \frac{n}{\rho \theta}$$

as in the case of no diffusion.

Hence our Ricardian assumptions yield starkly Ricardian results. If research productivity differences exceed barriers to trade in goods, countries specialize in research according to comparative advantage. But differences in research productivity fall short of barriers to trade in goods, countries do their own research even though they make use of each other’s ideas. Knowledge flows rather than good flows are how countries benefit from each other’s innovations.

What about the more realistic scenario in which ideas cross borders, but only with delay? Special situations without trade and with costless trade deliver insight, although the results are not as clean as above, and require numerical simulation.

4.4.3 No Trade

Consider the case in which trade barriers are so high that there is no trade ($d \to \infty$). This was implicitly the assumption in the model used to estimate innovation and diffusion among the top five research economies EK(1999).\footnote{If the South is very large compared to the North ($\lambda$ near 0), the value of $r_N$ in expression (10) can exceed one. In this case the North would specialize completely in research ($r_N = 1$). Depending on parameter values, the South might find research worthwhile as well. We leave this case as an exercise for the reader.}

5.5.6 The model in EK (1999) differs in several respects. For one thing, ideas in that paper are about inputs, which are not traded. But costless trade in final output allows for unbalanced royalty payments. For another, that
Without trade in goods, royalty payments need to balance, so that \( Y_n = Q_n, n = N, S \). We can also set: \( \pi_{nn} = T_n/(T_n + T_C), \pi_{ni} = 0, n \neq i \), and \( T_{nC} = T_C/(T_n + T_C) \). The value of ideas (8) then becomes:

\[
V_i = \frac{1}{\rho \theta - n} \frac{w_i L_i (1 - r_i)}{T_i + T_C}
+ \left( \frac{1}{\rho - n \theta} - \frac{1}{(\rho + \epsilon) \theta - n} \right) \frac{w_j L_j (1 - r_j)}{T_j + T_C}.
\]  

(11)

for \( i = N, S; i \neq j \). The first term on the right-hand side represents the value of the idea at home and the second its value abroad. Since goods aren’t traded, diffusion into the common pool has no implication for the value at home, while any return from abroad must await diffusion.

In steady state, balanced trade in royalties implies that:

\[
\frac{T_N}{T_N + T_S} \frac{T_C}{T_S + T_C} w_s L_s (1 - r_s) = \frac{T_S}{T_N + T_S} \frac{T_C}{T_N + T_C} w_N L_N (1 - r_N).
\]  

(12)

Substituting expression (12) into (11) and solving for steady state values of \( T_N, T_S, \) and \( T_C \) gives conditions for research intensity in each country:

\[
\Lambda = \frac{r_N}{1 - r_N} \frac{Ar_N + N r_S}{DAr_N + \epsilon \theta r_S}
\]

\[
A = \frac{r_S}{1 - r_S} \frac{r_S + N Ar_N}{Dr_S + \epsilon \theta Ar_N}
\]

where:

\[
\Lambda = \frac{n}{(\rho \theta - n) D}
\]

\[
A = \frac{a_N \lambda}{\alpha_S}
\]

\[
D = (\rho + \epsilon) \theta - n
\]

\[
N = \frac{\epsilon}{n + \epsilon}
\]

paper allows for diminishing returns to research activity at the national level. Diminishing returns are needed to reconcile observations on the small share of workers doing research and their apparently large contribution to productivity growth. Since our purpose here is a better understanding of the properties of the basic model rather than a realistic application to data, we do not introduce this complication here. Finally, the model allows for imperfect protection of innovations, both at home and abroad.
There is no analytic solution for the general case, although with $\epsilon = 0$ we of course get back to $r_N = r_S = n/\rho \theta$. Imposing symmetry ($A = 1$) we get the following expression for the ratio of research workers to non research workers:

$$\frac{r}{1-r} = \frac{n}{\rho \theta - n} G$$

where:

$$G = \frac{(\rho + 2\epsilon)\theta - n \ n + \epsilon}{(\rho + \epsilon)\theta - n \ n + 2\epsilon}$$

Note that $G$ is the product of two fractions, the first exceeding one and the second less than 1. If $G = 1$ we are back to $r = n/\rho \theta$. The first fraction reflects the added opportunities for earning royalties abroad that diffusion allows, which increases the incentive to do research. The second reflects the fact that diffusion means that foreign ideas compete with domestic ones at home, reducing research incentives. Which effect dominates depends on particular parameter values. More diffusion means more research if $n/\rho \theta$ exceeds $1/(1 + \theta)$ and vice versa. Simulations suggest that deviations from $r = n/\rho \theta$ are small regardless. Note that, consistent with the previous case, as $\epsilon \to \infty$, $G = 1$, so that, again, $r = n/\rho \theta$.

What about the role of country size, as measured by $A$? Using this same intuition, researchers in a larger country face less competition from foreign ideas, but have a smaller foreign market in which to earn royalties. In fact, our simulations reveal that the direction of the effect of country size on research intensity depends on parameter values.

We solve numerically for research intensity for the parameter values:

$$n = .01$$
$$\rho = .02$$
$$\theta = 8$$
$$\epsilon = .02$$
$$\alpha_N \lambda / \alpha_S = 1$$

In the symmetric case research intensity in each country is .057 (compared with .063 with no diffusion). We find that increasing the relative size of the North by a factor of 10 leads research activity there to increase to a labor share of: .062 while the share in the South falls to .051.
Raising $n$ to .02, however, reverses the effect of size on research activity (although deviations from symmetry are slight).

### 4.4.4 Costless trade

Say that trade is frictionless, meaning that $d = 1$. Case 3, with each country producing its own goods with the common technology, is, of course, eliminated as a possibility. If parameter values leave us in case 2 above, the wage in the two economies is identical, as are the value of ideas. If the return to research in the South is lower, only the North will undertake research. If the return is the same the location of research is indeterminate. In either case the world share of labor force engaged in research is the same as the closed economy value of $n/\rho \theta$.

More interesting is a set of parameter values that leave us in Case 1 above, with the Northern wage above the South’s. In this case the North uses only its own exclusive technology, but earns royalties from technology that has diffused into the common technology and is used in the South. The expression for the value of ideas, (8) above, simplifies by recognizing that spending on goods produced using the Northern technology is the same as spending on goods produced by Northern workers. Hence:

$$\pi_N Y_{Nt} + \pi_S Y_{St} = Q_N = \frac{1 + \theta}{\theta} w_N L_N (1 - r_N)$$

Only the South uses commonly available ideas, which it uses alongside its exclusive ideas. Total demand for Southern labor is thus:

$$(\pi_{NC} + \pi_{NS}) Y_{Nt} + (\pi_{SC} + \pi_{SS}) Y_{St} = Q_S = \frac{1 + \theta}{\theta} w_S L_S (1 - r_S)$$

$$= \frac{T_C + T_S}{T_C} \pi_{NC} Y_{Nt} + \pi_{SC} Y_{St}$$

Substituting these expressions into the value of ideas (8), and solving for the levels of $r_N$ and $r_S$ provides the following expressions:

$$r_N = \frac{n + \epsilon}{\rho \theta + \epsilon [1 - \theta (\alpha_N / \alpha_S)(w_S / w_N)]}$$

$$r_S = \frac{n}{\rho \theta} - \frac{\epsilon}{n + \epsilon} (\alpha_N \lambda / \alpha_S) r_N \left(1 - \frac{n}{\rho \theta}\right)$$

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To solve for the relative wage we need to refer back to our solution (3) for the static case above, setting \( d = 1 \) and replacing \( L_i^P \) with \( L_i(1 - r_i) \). Solving out for \( r_S \) we are left with:

\[
\frac{w_N}{w_S} = \left[ \frac{r_N}{1 - r_N} \frac{(\alpha_N/\alpha_S) \rho \theta - n}{\lambda (n + \epsilon)} \right]^{1/(1+\theta)}.
\]

(14)

For us really to be in case 1 requires, of course, that \( w_N/w_S > 1 \). This solution also requires that \( r_S \) exceed zero. For many parameter values, the South will end up doing no research at all. We don’t explore this case further, focusing on situations in which both countries continue to do research.

Note that \( r_N \) and the wage ratio \( w_N/w_S \) can be solved in terms of each other, with \( r_S \) determined as a function of \( r_N \). Note also that in the case of no diffusion we get the outcome \( r_N = r_S = n/\rho \theta \). For each country. With diffusion, more Northern research lowers how much goes on in the South below this level. At the same time, a graphical analysis of the system establishes that, for reasonable parameter values, an increase in diffusion from 0 raises research effort in the North. Hence introducing diffusion shifts research activity from the South to the North, in line with comparative advantage.

Solving the model numerically, an interesting base case emerges with the values:

\[
\begin{align*}
n &= .01 \\
\rho &= .02 \\
\theta &= 8 \\
\alpha_N/\alpha_S &= 5 \\
\lambda &= .1
\end{align*}
\]

With no diffusion (\( \epsilon = 0 \)) a fraction .0625 of workers in each country pursue research, and the relative wage is 1.48 times higher in the North. Raising diffusion so that \( \epsilon = .005 \) raises the share of researchers in the North to .175 and lowers it in the South to .035. The Northern wage advantage rises to a factor of 1.68. Taking into account the higher productivity of researchers in the North along with the smaller number of workers there, the effective level of research in the world rises by 30 percent.

Raising the relative size of the North to \( \lambda = .2 \) brings research in the two countries much closer to their autarky values, as does reducing the North’s productivity advantage.
in research, setting $\alpha_N/\alpha_S = 3$. Even if the North has no productivity advantage in doing research ($\alpha_N = \alpha_S$), it will do more research than the South in steady state (.083 vs. .062) with the other parameters as above.

5 Conclusion

What does our analysis suggest about the implications of globalization, either in the form of greater diffusion of ideas or lower trade barriers, for research incentives? And what role does country size play? In our base case with no diffusion, countries engage in the same amount of research regardless of their relative size and research productivity. More research productive countries are of course richer, since the same research effort yields higher returns.

Jumping to a world with instantaneous diffusion can have a major effect on the allocation of research activity, or none at all, depending on the importance of trade barriers relative to differences in research productivity. When comparative advantage in research is more pronounced, instantaneous diffusion leads to Ricardian specialization in research activity. But if trade barriers are more significant, countries continue to do the same amount of research as with no diffusion. Given the amount of diffusion, a lowering of trade barriers can lead to more specialization in research.

Intermediate levels of diffusion imply less knife-edged results. Under plausible parameter values we find some tendency for greater diffusion to shift research toward countries with greater research productivity.

While more trade and diffusion may cause research activity to shift across countries, our analysis provides little to suggest that it will increase research effort overall. Globalization, either in the form of lower trade barriers or more rapid diffusion, provides researchers larger markets, but also exposes them to more competition. In the case of no diffusion, a lowering of trade barriers has no effect on research incentives, as these forces exactly balance. With diffusion results are less clear cut, but the effects of globalization in either form turn out to
be ambiguous in sign and, in our numerical examples, quantitatively small. In terms of global research activity, the closed economy result, that worldwide research intensity depends on the ratio of the population growth rate to the discount factor and the markup parameter, seems not far off.
References


