1. (40 points) The gravity equation for bilateral trade says that country $n$’s purchases from country $i$ satisfy $X_{ni} = Y_n Y_i / \tau_{ni}$ where $Y_n$ is GDP in $n$ and $\tau_{ni}$ is an increasing function of the cost of moving goods from $i$ to $n$. In what follows assume there are $N$ countries. For simplicity ignore intermediates so that labor is the only input, country $i$ having $L_i$ available.

(a) Theoretical derivations of the gravity equation often work only under the assumption of no trade costs. In that case, derive the gravity equation from the monopolistic competition model of trade.

(b) Consider the opposite extreme in which countries have equal GDP’s (but not equal labor forces) and iceberg trade costs are positive (common across all country pairs). In that case, derive the gravity equation from the Ricardian model in TGT (“Technology, Geography, and Trade”). Hint: you need to find the relationship between $T_i$ and $L_i$ that will endogenously yield a common value of GDP.

(c) Now consider general versions of either monopolistic competition or TGT. Set $\theta = 8$ in TGT or equivalently $\sigma = 9$ in the monopolistic competition model. The United States spends $2376$ billion on manufactures of which $319$ billion is imported, including $9$ billion from France. France spends $422$ billion of which $104$ billion is imported, including $8$ billion from the United States. (These are the actual numbers for manufactures in 1986.) What can you infer, or what bounds can you place, on the iceberg cost of moving goods from France to the United States and the cost of moving goods from the United States to France?

(d) Suppose each individual producer in a country produces exactly 1 good. Under this assumption, what does TGT imply for the fraction of US producers exporting to France and the fraction of French producers exporting to the United States? Explain how this prediction differs from the monopolistic competition model.
2. (40 points) Consider the Heckscher-Ohlin model with a home country \( H \) a foreign country \( F \), two goods \( X \) and \( Y \), and two factors \( S \) (skilled labor) and \( L \) (unskilled labor). Make the following assumptions: the technology is

\[
Q_j = \min \{ S_j/a_{Sj}, L_j/a_{Lj} \} \quad j \in \{X,Y\},
\]

the endowments satisfy

\[
\frac{\bar{S}_F}{\bar{L}_F} < \frac{\bar{S}_H}{\bar{L}_H},
\]

and preferences in each country are \( U = C_X^\beta C_Y^{1-\beta} \). Let good \( Y \) be the numeraire, i.e. \( P_Y = 1 \).

(a) What conditions on \( P_X, \bar{S}_H, \) and \( \bar{L}_H \) are needed for home to be producing both goods? Under these conditions, holding \( P_X \) fixed, what happens in the home country if more people go to college and become skilled labor? What theorem does this illustrate?

(b) Under the conditions above, holding \( P_X \) fixed, suppose that \( a_{LX} \) and \( a_{LY} \) each fall by \( x \) percent (with home remaining diversified). Show that this change can explain one of the following facts but gets the other one wrong: (i) a rise in the skill wage premium and (ii) a rise in skill intensity within each industry.

(c) What are minimal restrictions on endowments and or preferences that guarantee factor price equalization? Show how your answers to the two questions above are modified in general equilibrium.

(d) Now, suppose \( H \) is twice as productive as \( F \) in producing either good. What are the conditions for equalization of the price of effective factors? Write out the Vanek equations in terms of the parameters.