ECON 8002/4162: Microeconomic Analysis
PROBLEM SET #3
(due in the TA session, 11/15)

These questions are motivated by the first part (pp. 322-323) of the Lucas (1967) paper. I will use the same notation and will (sometimes implicitly) make the same assumptions (unless stated otherwise).

1. Consider a discrete-time version of the problem, so that the value of the firm at date zero is

$$V_0 = \sum_{t=0}^{\infty} (1+r)^{-t} R_t.$$

The capital accumulation equation is

$$K_t = I_t + (1-\delta)K_{t-1},$$

with K_{-1} given.

   (a) Show that $K_t = (1-\delta)^{t+1}K_{-1} + \sum_{s=0}^{t}(1-\delta)^{t-s}I_s$.

   (b) Using the expression above, write down the objective function of the firm at date zero in terms of $\{L_t\}$ and $\{I_t\}$, given K_{-1}. What are the FOC's for maximizing the objective function?

   (c) Use the FOC's for L_t, $(t = 0, 1, \ldots)$ to prove that the marginal product of labor will be a constant (this is easy). Use the FOC's for I_t, $(t = 0, 1, 2, \ldots)$ to prove that the marginal product of capital will be a constant (this requires some work).

   (d) Show that the firm will act as if it is simply solving the following static maximization problem at each date $t = 0, 1, 2, \ldots$:

$$\max_{K_t, L_t} [pF(L_t, K_t) - wL_t - \frac{r+\delta}{1+r} vK_t].$$

2. Now, return to the continuous-time problem as it appears in the paper. The standard approach for solving the firms optimization problem is to write down the Current Value Hamiltonian:

$$H(t) = R(t) + \lambda(t)\dot{K}(t),$$

where $\lambda(t)$ acts somewhat like a series of Lagrangian multipliers. You want to substitute out the constraint for $\dot{K}(t)$. The optimal solution solves the following FOC's:

$$\frac{\partial H(t)}{\partial L(t)} = 0,$$

$$\frac{\partial H(t)}{\partial I(t)} = 0,$$

and

$$\frac{\partial H(t)}{\partial K(t)} = r\lambda(t) - \dot{\lambda}(t).$$

(There is one more condition, the transversality condition, which will be automatically satisfied for this problem.)

Use the Hamiltonian apparatus to prove the result on page 323 that Lucas attributes to Haavelmo (1960).