

ECON 8002/4162: Microeconomic Analysis

Final Exam

12/18/2001

1. Suppose two firms can each produce some good at a unit cost c . If the lowest price is P , the quantity demanded of the good is $P^{-\beta}$, with $\beta > 1$.
 - (a) Calculate the total quantity demanded under Bertrand competition, under Cournot competition, and under monopoly (say the firms merge).
 - (b) In any of the three cases above is it possible to relax the assumption of $\beta > 1$? If so, what restriction is really necessary in each case in order for the result to make sense?
 - (c) Suppose that if a firm enters, it incurs a fixed cost $F \geq 0$ (whether or not it ends up producing anything). First firm 1 decides whether to enter, next firm 2 decides whether to enter, and finally entrants compete according to either Bertrand or Cournot. Write down the extensive form of the game (including payoffs) for the version in which firms choose quantities in the final stage.
 - (d) Continuing with the entry game, assume that the solution is a sub-game perfect Nash equilibrium. Derive a range for F such that consumers prefer that in the final stage of the game firms choose quantities rather than prices. Why is the result so different from the special case of $F = 0$?
2. According to Jensen's inequality, if X is a nondegenerate random variable and $f(X)$ is a strictly concave function, then $E[f(X)] < f(E[X])$.
 - (a) A degenerate random variable is one that always takes on the same value. Why does the statement of Jensen's inequality require X to be nondegenerate?
 - (b) Prove Jensen's inequality.
3. Consider Krugman's adaptation of the Dixit-Stiglitz model. The utility of an individual is $U = \sum_{i=1}^{\bar{n}} c_i^\theta$, with $0 < \theta < 1$. Assume \bar{n} is very large, so that the actual number of varieties supplied n is less

than \bar{n} . The production technology is $l_i = \alpha + \beta x_i$. The wage can be normalized to 1. If the labor force is L then $x_i = Lc_i$.

- (a) What are the equations for marginal cost and average total cost? Draw a graph showing marginal cost and average total cost as a function of x .
- (b) Show that in the monopolistically competitive equilibrium, $x_i = \alpha\theta/[\beta(1-\theta)]$ and $n = (1-\theta)L/\alpha$.
- (c) Consider country 1 with a labor force L_1 in a world with a labor force L . Remember that with free trade it is as if everyone in the world is living in one big country with a labor force of L . Calculate the percentage gain in utility to an individual in country 1 if the country goes from no trade with the rest of the world to free trade with the rest of the world. Explain the way in which your result depends on θ .

4. Define a Nash Equilibrium in mixed strategies.