Optimal Unilateral Carbon Policy*

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Abstract

We consider climate policy in a world with international trade where one region imposes a climate policy and the rest of the world does not. Regional climate policy may generate inefficient shifts in the location of extraction, production, and consumption, an effect known as leakage. We derive the optimal unilateral policy and show how it can be implemented through taxes. The optimal policy involves (i) a tax on extraction at a rate equal to the marginal harm from emissions, (ii) a border adjustment on the import and export of energy and on the import, but not the export, of goods, with the border adjustment at a different (usually lower) rate than the extraction tax rate, and (iii) an export policy designed to expand the export margin. The optimal policy controls leakage by controlling the price of energy and exploits international trade to expand the reach of the climate policy. We calibrate and simulate the model to illustrate how the optimal policy compares to more traditional policies.

Keywords: carbon taxes, border adjustments, leakage
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1 Introduction

Global negotiations have given up trying to achieve a uniform approach to climate change, such as a harmonized global carbon tax. Instead, current negotiations focus on achieving uniform participation, with each country pursuing its own approach and its own level of emissions reductions. As a result, policies to control emissions of carbon dioxide vary widely by country, and are likely to continue to do so for the indefinite future.

Widely varying carbon policies potentially affect patterns of trade, the location of extraction, production and consumption, the effectiveness of the policies, and the welfare of people in various countries or regions. These effects are of critical importance to the design of carbon policy and to its political feasibility. For example, trade and location effects were central to the design of the European Union Emissions Trading System, the Regional Greenhouse Gas Initiative, and California's carbon pricing system. One of the reasons that the United States did not ratify the Kyoto Protocol was concern about the lack of emissions policies in developing countries and the resulting trade effects. Unless concerns about the effects of differential carbon prices are addressed, it may be difficult to achieve significant reductions in global emissions.

To address this problem, we develop an analytic general equilibrium model of carbon pricing and trade, where one region (Home) imposes a carbon policy and the rest of the world (Foreign) does not. The model is a mix of Markusen (1975) and Dornbusch, Fisher, and Samuelson (1977; henceforth DFS). Following Böhringer, Lange, and Rutherford (2014), we restrict policies adopted by Home to those that do not make Foreign worse off. Our solution strategy borrows from Costinot, Donaldson, Vogel, and Werning (2015; henceforth CDVW). We solve for the outcomes that are optimal for Home and then show how those outcomes can be implemented in a decentralized equilibrium using taxes and subsidies.

Our solution to the model suggests a novel approach to the problem of unilateral carbon pricing, one where increasing the extent of trade actually improves outcomes rather than making them worse. The approach combines four elements: (i) a domestic carbon tax on the extraction of fossil fuels at a rate equal to the marginal harm from emissions; (ii) a border adjustment on imports and exports of energy at a lower rate than the extraction tax rate (which we will call a "partial" border adjustment); (iii) a border adjustment, at that same partial rate, on imports but not exports of goods; and (iv) an export subsidy designed to expand low-carbon exports from the carbon-pricing region to the rest of the world.

The partial border adjustment on energy shifts a portion of the extraction tax downstream to production. Therefore, the policy combines a tax on extraction and on production. The border adjustment on imports of goods ensures that foreign producers selling goods in the taxing region face the same tax as domestic producers. The export subsidy is not a rebate of prior taxes paid, as in a conventional border adjustment. Instead, it expands exports via fixed subsidies not determined by prior taxes. Combined, the taxing region maximizes the reach of its carbon tax to include all goods produced domestically (regardless of where they are consumed), all goods consumed domestically (regardless of where they are produced), and, moreover, expands the scope of its exports to further broaden the tax base. As the extent of trade increases, the taxing region is able to expand the tax base further, generating better outcomes. Moreover, by combining extraction taxes and production taxes, the taxing region is able to control the price of energy seen by foreign actors, thereby minimizing leakage-like effects. The policy has elements of Markusen (1975), who suggested combining supply-side and demand-side taxes, and of Fisher and Fox (2012), who suggested per-unit subsidies rather than rebates for exports.

To understand the quantitative implications of our analysis, we calibrate the model and solve it numerically. In our core calibration, we assume that the OECD countries impose a carbon price and the rest of the world does not. We compare the optimal policy to more conventional policies: (i) a tax only on the extraction of fossil fuels (as suggested by Metcalf and Weisbach (2009)); (ii) a tax only on the use of energy in production (which is how most cap and trade systems work); and (iii) a tax on production combined with conventional border adjustments (which is the structure of many recent carbon tax proposals). Conventional border adjustments shift a tax on the use of energy in domestic production to a tax on the energy embodied in domestic consumption, so we think of (iii) as a tax only on implicit consumption of carbon. We also examine the three pairwise combinations of the conventional policies: a hybrid of an extraction tax and a production tax, a hybrid of an extraction tax and a consumption tax, and a hybrid of a production tax and a consumption tax, in each case choosing the optimal mix of the two taxes.

Two of the hybrid taxes perform particularly well in our simulations, the hybrids involving an extraction tax. These two policies do not perform quite as well as the optimal tax, but considerably outperform the more standard production and consumption taxes. The combination of an extraction tax and a production tax would also be much simpler to impose than the other choices. It could be imposed with a nominal tax on extraction combined with border adjustments (at a lower rate) on the imports and exports of energy, but not goods. As suggested by Metcalf and Weisbach (2009), an extraction tax would be easy to impose because there are a relatively small number of large extractors who would need to remit taxes. Border adjustments on energy would also be easy to impose because imports and exports of energy are already carefully tracked. As a result, the simulations suggest that the combination of an extraction tax and a production is a promising policy to explore.[1]

The intuition for why these hybrids and the optimal tax perform so much better than conventional taxes is that the hybrids moderate the effects of the policy on the price of energy seen by actors in the non-taxing region. Taxes on emissions from production or those same taxes with border adjustments reduce the demand for energy, lowering the global price. This reduction in the global price of energy creates incentives to expand production and consumption in non-taxing regions. Taxes on extraction reduce domestic extraction, and, therefore, raise the price of energy in non-taxing regions. As a result, they induce an increase in extraction in non-taxing regions. The two extraction tax hybrids and the optimal tax moderate the change in the price of energy by combining demand-side taxes with a tax on the supply of energy. This allows the taxing region to control the effects of its policies on the activities in the non-taxing region. The mix of the taxes, as determined by the border adjustment (relative to the underlying extraction tax), takes into account the wedges seen by Foreign extractions, producers, and consumers between the optimal price of energy and the price they face, balancing tax-induced increases in Foreign extraction with increases in Foreign production and consumption.

Our core model does not include renewable energy, and stimulating renewables is often seen as a central goal of carbon pricing. To examine this issue, we extend the analysis to show that including renewables only requires modest adjustments to the optimal policy. Not surprisingly, renewables are exempt from the tax on extraction. If they can be sold in the market at

[1]While we do not address legal issues, it is also likely that the extraction/production hybrid raises fewer concerns about WTO compatibility than do the optimal tax or conventional border adjustments imposed on goods.

the same price as fossil fuels, this exemption stimulates the production of renewables. In addition, while the optimal policy attempts to limit increases in fossil fuel extraction in other countries, it does not do so for renewables because the additional use of renewables in other countries does not generate harm.

The paper proceeds as follows. The remainder of this section provides additional motivation and reviews the relevant literature. Those eager to get to the foundations of our analysis may skip to Section 2, which lays out the basic elements of the model and characterizes the competitive equilibrium absent taxes. Section 3 solves the problem of a planner designing an optimal carbon policy for one region with the other region behaving as in the competitive equilibrium. In Section 4 we derive a set of taxes and subsidies that implement the optimal policy, which we take to be the policy recommendations of this analysis. We explore the quantitative implications of the optimal policy in Section 5, using a calibrated version of the model. Section 6 extends the analysis to include a renewable energy sector. Section 7 concludes.

1.1 Background

As noted, the possibility that varying carbon prices in different regions might affect patterns of trade, the location of extraction, production, or consumption, and the effectiveness of carbon prices has been a central concern in the design of carbon policy. Our central motivation is to understand these effects and the optimal response to them. A related motivation is that there is a virtual zoo of possible responses, and, while there has been extensive work analyzing some of the most prominent responses, so far it has not been clear how to pick among the full range. That is, the question is not simply whether adding border adjustments to a conventional production tax is desirable, which is the focus of much of the literature. Instead, there are a wide variety of policies, and we need a method of picking among them. To illustrate this latter problem, we describe the range of possibilities here, along the way defining terms that will be useful in understanding the optimal, decentralized solution.

For simplicity, we assume (here and throughout the paper) that the price on carbon is imposed via a tax rather than a cap and trade system. Although there may be differences between taxes and cap and trade systems (e.g., Weitzman (1974)), these differences are not relevant for our purposes. We also only focus on carbon emissions from fossil fuels, which have been the central focus of existing and proposed carbon prices, ignoring agriculture anddeforestation, two other major sources of emissions. For the most part, we also set aside administration concerns, including the problems of imposing border adjustments discussed in Kortum and Weisbach (2017). Doing so allows us to understand the shape of the optimal policy, which is a necessary step to designing administrable policies.

Normally, Pigouvian taxes need to be imposed directly on the externality-causing activity rather than on imperfect proxies. There is, however, an almost one-for-one relationship between fossil fuel inputs into the economy and eventual emissions. That is, almost all carbon molecules that enter the economy as fossil fuels are eventually emitted as CO_2 through combustion. This fact means that in a closed economy we can tax carbon at any, or multiple, stages of production without losing accuracy.

Metcalf and Weisbach (2009) exploit this fact to suggest imposing a carbon tax upstream on the extraction of fossil fuels. They reasoned that there are a small number of large, sophisticated extractors, compared to a much larger number of manufacturers using fossil fuels and a vastly larger number of consumers of products made using fossil fuels. They estimated that the United States could tax essentially all domestic extraction of fossil fuels by taxing just 2,500 entities, compared to, say, the roughly 250 million vehicle tailpipes, among many other items, that would have to be taxed with a direct tax on consumers.

In a closed economy, a tax on extraction would be the same as a tax anywhere else in the chain of production (but for administrative costs). A tax on extraction would be embedded in the price of the fuel, causing manufacturers and consumers (as well as extractors) to internalize climate externalities. This is not true, however, in an open economy. Extraction taxes increase the pre-tax price of fossil fuels. If t_e is the extraction tax and p_e the pre-tax price of energy, extractors receive $p_e - t_e$ after tax. Unless the tax is entirely borne by extractors, p_e will go up. Because the price of fossil fuels goes up, extraction taxes cause foreign extractors, not subject to the tax, to increase extraction, generating what we call extraction leakage. Extraction leakage reduces the effectiveness of an extraction tax. To the extent Foreign emissions go up because of extraction leakage, the taxing region suffers harm that it might otherwise have avoided.

While extraction taxes cause a shift in where extraction occurs, on their own they do not shift where production and consumption occur. If there is a global price for energy, all producers and all consumers, globally, see the same higher price for energy generated by the extraction tax in the taxing region. They all adjust their production and consumption accordingly, with no particular differentiation between actors in the taxing region and the non-taxing region.

Actual carbon prices are usually imposed on emissions from domestic production—that is, on the smokestack—rather than on extraction. For example, the European Union Emissions Trading System is on emissions from industrial use of fossil fuels.[2]

With a production tax at rate t_p, producers pay $p_e + t_p$ for energy, increasing the after-tax price of energy. Once again, in a closed economy, the effects of taxing production would be the same as taxing extraction. In an open economy, however, their effects will not be the same. Production taxes lower the global price of energy because demand will go down: producers will shift to cleaner manufacturing techniques and consumers will demand fewer energy-intensive goods.

To the extent there is a global price of energy, all extractors, globally, see a lower price of energy and extract less. There is no extraction leakage with a pure production tax. That is, shifting away from an extraction tax toward a production tax moderates extraction leakage by moderating the price-increasing effect of an extraction tax (an effect we will see in our optimal solution).

Production taxes however, cause production to shift to untaxed regions because they reduce the comparative advantage of producers in the taxed region. This effect is known as production leakage, or because of the predominance of production taxes, often just leakage. Leakage is generally used as the central measure of the (in)effectiveness of a carbon policy. Fowlie (2009) called it the defining issue in the design of regional climate policies

If there were no trade costs, production taxes would not affect the location of consumption. All consumers, even those abroad, who purchase goods produced in the taxing region would face a higher price for those goods. And all consumers, even those in the taxing region, would see a lower price for goods produced abroad. Production taxes affect where goods are produced but not where they are consumed. With trade costs, however, taxes on production may shift where consumption takes place because trade costs tie

[2]Some recent proposals in the United States require extractors rather than producers to remit taxes. The taxes, however, still fall on domestic extraction because they impose taxes on imports of fossil fuels and rebate taxes on exports of fossil fuels.

production and consumption together to some extent.

Finally, a carbon tax can be imposed directly on consumption. A tax on consumption would be based on the emissions associated with each good when it was produced. For example, if a consumer buys a toaster, the consumer would pay a tax based on the emissions from the production of the toaster. Because of the very large number of products and consumers, and the difficulty of determining the tax, carbon taxes are not normally proposed to be imposed this way. Gasoline taxes, however, might be thought of as a version of a consumption tax, and these are collected at the pump.

These three "pure" taxes, can be combined. For example, a country that wants to impose a \$100/ton tax on emissions of CO_2 could impose a \$50/ton tax on extraction, a \$30/ton tax on carbon used in production, and a \$20/ton tax on consumption. As we will suggest, the right mix allows the country to moderate the effects of each of the pure taxes. For example, imposing both an extraction tax and a production tax can balance the negative effects on the location of extraction that arise from a pure extraction tax with the negative effects on the location of production from a pure production tax.

The last piece of terminology is "carbon border adjustments" or simply border adjustments.[3] Border adjustments are taxes on imports or rebates of prior taxes paid on exports. They can apply to either fossil fuels or goods, or both. For fossil fuels, the border adjustment is on the carbon content of the fossil fuel. For goods, the border adjustment is on the carbon emissions from the production of the good, what we call the embodied carbon or embodied energy. Kortum and Weisbach (2017) provide a more detailed description of border adjustments.

Border adjustments shift the tax downstream. For example, an extraction tax with border adjustments on the import and export of fuels becomes a tax on domestic production. Any fuel that is extracted domestically but exported has the tax rebated, and any fuel that the country extracted abroad but imported has a tax imposed. All fuel used domestically, and only that

[3]The term "border adjustment" is most often used in connection with destination-based VATs, widely used throughout the world. Border adjustments in this context are rebates of prior VAT paid when a good is exported and the imposition of VAT when a good is imported.

The term "carbon border adjustment" is a border adjustment based on the carbon content of goods including the carbon emitted during production, rather than their value (as in a VAT). For simplicity, we shorten the term to just "border adjustment" because the usage is unambiguous here.

fuel, bears a tax. Therefore, we can equivalently impose an extraction tax plus a border adjustment or a production tax. They differ only in their nominal description. Similarly, a border adjustment on imports and exports of goods shifts the tax from production to domestic consumption, and we can equivalently impose a production tax plus a border adjustment or a consumption tax.

Full border adjustments are imposed at the same rate as the underlying tax (e.g., a \$100 carbon ax on domestic production would have a \$100 tax imposed at the border). Border adjustments can be "partial" in that they are imposed at a different rate than the underlying tax. If the rate is less than the underlying tax, we can think of the border adjustment as shifting that portion downstream. For example, if a nominal extraction tax is imposed at $t_e = \$100/\text{ton}$ of CO_2, and the border adjustment is at $t_b = \$75/\text{ton}$, we can think of this as an effective tax on extraction of $\tilde{t}_e = \$25/\text{ton}$ tax on extraction and a tax of $\tilde{t}_p = \$75/\text{ton}$ tax on production. Therefore, we can implement combinations of the three pure taxes via nominal taxes and border adjustments imposed at different rates than the underlying tax.

Border adjustments can also apply differently to imports and exports. For example, they can be applied to imports but not exports. A production tax with a border adjustment applied only to imports becomes a tax on all domestic production and on all domestic consumption generating a broader tax base than any of the three pure taxes. More generally, the border adjustment can be applied at different rates to imports and exports (and both those rates might be different than the rate of the underlying tax). We can decompose the effects in the same way as suggested above.[4]

As can be seen, there are a large number of possible taxes. Our goal is to understand the optimal mix of these possibilities for taxing regions.

[4]Border adjustments might apply only to a subset of goods, such as only to goods that are particularly energy intensive. Many border adjustment proposals are limited in this way, in large part to minimize administrative costs. Modern economies import and export a vast number of different goods, and computing accurate border adjustments for each of these goods would be difficult. By imposing border adjustments only where their effects are likely to be large, the administrative costs can be reduced. Because we abstract away from implementation costs, we do not consider this type of border adjustment.

1.2 Prior Literature

Because of its prominence, there is a voluminous prior literature studying this problem. The overwhelming majority of studies use computable general equilibrium models to simulate carbon taxes and border adjustments. By our count, there are over 50 CGE studies of the general problem of differential carbon prices in the peer-reviewed literature (and many more in the gray literature) and each study considers multiple different scenarios, which means that there are hundreds of simulations of the problem.[5] For example, Branger and Quirion (2014) perform a meta-analysis of 25 studies of differential carbon taxes (20 of which were CGE studies, 5 of which were partial equilibrium studies). These 25 studies, which make up only a portion of the literature, had 310 different modeled scenarios.

CGE studies almost uniformly use leakage as their measure of the effects of differential carbon prices. Leakage is commonly defined as the increase in emissions in non-taxing regions as a percentage of the reduction in emissions in the taxing region (Hence, 100% leakage means the policy is totally ineffective in reducing global emissions). Leakage estimates fall within a relatively consistent range. The Branger and Quirion meta-study finds leakage rates between 5% and 25% without border adjustments. They also find that border adjustments reduce leakage by about a third to be within a range of 2% to 12% with a mean value of 8%. Similarly, the Energy Modeling Forum commissioned 12 modeling groups to study the effects of border adjustments on leakage using a common data set and common set of scenarios. Bohringer et al. (2012). They considered emissions prices in the Kyoto Protocol Annex B countries (roughly the OECD) that reduce global emissions by about 9.5%. Without border adjustments, leakage rates were in the range of 5% to 19% with a mean value of 12%. They also find that border adjustments reduce leakage by about a third, with a range between 2% and 12% and a mean value of 8%. Elliott et al. (2013) replicated 19 prior studies within their own CGE model, finding leakage rates between 15% and 30% for a tax on Annex B countries that reduced global emissions by about 13%.[6]

[5] For surveys of the leakage literature, see Droge et al. 2009, Zhang 2012 and Metz et al. 2007

[6] A smaller number of studies focus on the effects of carbon taxes on particular energy-intensive and trade-exposed sectors. For example, Fowlie et al. (2016) consider the effects of a carbon price on the Portland cement industry. They find that a carbon price has the potential to increase distortions associated with market power in that industry. Leakage compounds these costs. They find that border adjustments induce negative leakage because

We use an analytic general equilibrium model of trade to study the problem. This approach allows us to uncover the underlying economic logic for why some policies perform better than others, although it means that our quantitative analysis is more illustrative than definitive because the model is stripped down. There are a small number of studies that precede us in this approach. The classic study, which we build on, is Markusen (1975). Markusen analyzes a two-country, two-good model in which production of one of the goods generates pollution that harms both countries. Writing before climate change was a widespread concern, he considers a simple pollutant, such as the release of chemicals into Lake Erie by polluters in the United States, which harms Canada (as well as the United States). One of the countries imposes policies to address the pollution; the other is passive. Markusen finds that the optimal tax is a Pigouvian tax on the dirty good combined with a tariff (if the good is imported) or a subsidy (if it is exported). The optimal tariff or subsidy combines terms of trade considerations and considerations related to leakage and is generally lower than the Pigouvian tax.[7]

2 Basic Model

Two countries, Home and Foreign, are endowed with labor, L and L^*, as well as energy deposits E and E^*. The $*$ distinguishes Foreign from Home, whose carbon policy we seek to optimize.

Each country has three sectors: energy e, goods g, and services s. Energy is extracted from deposits using labor, goods are produced by combining labor and energy, and services are provided with labor only. Labor is perfectly mobile across the three sectors within a country.[8] As in DFS, goods come in a continuum, indexed by $j \in [0, 1]$.

of how industry actors respond, and can generate significant welfare gains at high carbon prices.

[7]Hoel (1996) generalizes Markusen's analysis and produces similar results in the context of climate change and carbon taxes. He also considers the case where the country may not impose tariffs. In this case, the optimal policy will involve carbon taxes that vary by sector (even though the harms from emissions do not vary by sector). There are a number of other analytic models of the problem, including Böringher, Lange and Rutherford (2014), Holladay et al (2018), Hemous (2016), Baylis et al. (2014), Jakob, Marschinski and Hubler (2013), Fischer and Fox (2012, 2011), and Hoel (1994).

[8]What we call "labor" can be interpreted as a combination of labor and capital used to extract energy, produce goods, and provide services.

2.1 Preferences

We denote services consumption by C_s and define an index of goods consumption by:

$$C_g = \left(\int_0^1 c_j^{(\sigma-1)/\sigma} dj \right)^{\sigma/(\sigma-1)},$$

where $\sigma > 0$ is the elasticity of substitution across the individual goods j. Preferences in Home are:

$$U = C_s + \eta^{1/\sigma} \frac{C_g^{1-1/\sigma} - 1}{1 - 1/\sigma} - \varphi Q_e^W,$$

where $\eta > 0$ governs Home's overall demand for goods. Home's marginal harm from global emissions is $\varphi > 0$, which multiplies global energy extraction, $Q_e^W = Q_e + Q_e^*$.[9] Preferences in Foreign are the same except with η^* in place of η, σ^* in place of σ, and φ^* in place of φ.[10]

2.2 Technology

Energy is deposited in a continuum of fields, characterized by different costs of extraction. The quantity of energy that can be extracted at a unit labor requirement below a is given by $E(a)$ in Home (for $a \geq \underline{a} \geq 0$) and $E^*(a)$ in Foreign (for $a \geq \underline{a}^* \geq 0$).[11] We assume efficient extraction within each region so that low cost fields are tapped first. The labor L_e employed in Home to extract energy Q_e satisfies:

$$L_e = \int_0^{\bar{a}} a \, dE(a), \tag{1}$$

[9] Prior to introducing multiple energy sources, including renewable energy, in Section 7, we equate energy with a homogeneous fossil fuel, measured by its carbon content.

[10] We follow Grossman and Helpman (1994) in adopting quasi-linear preferences, which greatly simplifies the analysis. To ensure that the marginal utility of income is 1 we assume $C_s > 0$ and $C_s^* > 0$, a condition which is easy to confirm. In the special case of $\sigma = 1$ the preferences are:

$$U = C_s + \eta \int_0^1 \ln c_j dj - \varphi Q_e^W,$$

and likewise in Foreign.

[11] We assume that $E(a)$ is a strictly positive, continuous, and strictly increasing function on $a \geq \underline{a}$ and that $E(a) = 0$ for $a < \underline{a}$. The same applies to $E^*(a)$, with $\underline{a}^*$ in place of $\underline{a}$.

and

$$Q_e = E(\bar{a}), \tag{2}$$

so that $\bar{a}$ is the highest-cost field that is tapped when Home extracts Q_e. The output of the energy sector is in turn used as an intermediate input by the goods sector.

Goods $j \in [0,1]$ are produced with input requirement a_j in Home using a Cobb-Douglas combination of labor and energy:

$$q_j = \frac{1}{\nu a_j} L_j^{\alpha} E_j^{1-\alpha}, \tag{3}$$

where L_j is the labor input, E_j is the energy input, $0 < \alpha < 1$ is the output elasticity of labor, and $\nu = \alpha^{\alpha} (1-\alpha)^{1-\alpha}$. The production function in Foreign is the same, but with a_j^* in place of a_j.[12]

Services, in quantitites Q_s and Q_s^*, are provided in both countries with a unit labor requirement. We take services to be the numéraire, with price 1.[13]

2.3 International Trade

We assume that energy and services are costlessly traded between Home and Foreign, with the relative price of energy denoted by p_e. This price will dictate outcomes in Foreign within the planning problem that we consider below.

Trade in the continuum of manufactured goods follows DFS. Goods are arranged in decreasing order of Home's comparative advantage:

$$\frac{a_j^*}{a_j} = F(j), \tag{4}$$

with $F(j)$ a strictly decreasing continuous function.[14]

[12]In line with our Ricardian assumptions, we treat α as common across goods and countries. Including the constant ν in the production function simplifies expressions for costs that will appear later. This technology is nearly identical to the production and pollution technology in Shapiro and Walker (2018), although α here is $1-\alpha$ there. They use it to assess the reduction of air pollution in US manufacturing from 1990-2008.

[13]We will assume that $Q_s^* > 0$ so that, given the unit labor requirement for services, the wage in Foreign is $w^* = 1$. This outcome is guaranteed with a large enough labor endowment in Foreign.

[14]In order to have well defined integrals in what follows, we also assume that a_j and a_j^* can be treated as continuous functions of j.

Goods are traded subject to iceberg costs on Home's exports $\tau \geq 1$ and on Home's imports $\tau^* \geq 1$. The total input requirement for Home to supply good j to Foreign is thus τa_j and for Foreign to supply good j to Home $\tau^* a_j^*$.

2.4 Labor and Energy Requirements

We now introduce a notation for energy and labor input requirements that will prove convenient throughout the rest of the paper. At a given energy intensity:

$$z_j = E_j / L_j$$

we can invert Home's production function (3) to get the unit energy requirement for good j:

$$e_j(z_j) = \nu a_j z_j^{\alpha}, \tag{5}$$

with corresponding unit labor requirement $l_j(z_j) = e_j(z_j)/z_j$. Unit energy and labor requirements in Foreign, $e_j^*(z_j)$ and $l_j^*(z_j)$, are the same but with a_j^* in place of a_j.[15]

So as not to constrain the optimal policy, the energy intensity for good j may depend not only on where the good is produced but also on where it is shipped. To handle that possibility requires some additional notation.

For each good j we distinguish between Home's exports, $x_j \geq 0$ and Home's production for consumption in Home, $y_j = q_j - \tau x_j \geq 0$. We also distinguish between Home's imports, $m_j \geq 0$ and Foreign's production for consumption in Foreign, $y_j^* = q_j^* - \tau^* m_j \geq 0$. (Note that we define exports and imports in terms of the quantity that reaches the destination.) For each good j we allow for the possibility of four different energy intensities z_j^y, z_j^x,z_j^m, and z_j^*, one for each of the four lines of production y_j, x_j, m_j, and y_j^*.[16]

2.5 Carbon Accounting

We take a unit of energy to be a unit of carbon. Energy can be extracted in both countries and Home may may either export or import energy from

[15] Our *unit energy requirement*, $e_j(z_j)$, is sometimes called *emissions intensity* in the environmental economics literature, e.g. Shapiro and Walker (2018). We instead use the term *energy intensity* for energy per worker, z_j, by analogy to the common use of *capital intensity* for capital per worker.

[16] Because Foreign can set z_j^* independently from how it sets z_j^m, we do not include a so-called Brussels effect, as suggested by Bradford (2020).

Foreign. Carbon is released when the energy is used to produce goods. These goods, embodying carbon emissions, may be traded before being consumed by households. We can therefore trace carbon from its extraction through its release into the atmosphere and finally to its implicit consumption.

We define G_e as total intermediate demand for energy by the goods sector in Home and G_e^* by that of the goods sector in Foreign. Home's net exports of energy, $Q_e - G_e$, may be positive or negative. These expressions account for the first level of trade in carbon.

The second level of trade in carbon is embodied in goods. Table 1 depicts the bilateral flows, with rows indicating the location of consumption and columns the location of production. For example, Home's implicit consumption of carbon C_e (in the upper right) is the sum of carbon released by producers in Home serving the local market, C_e^{HH}, and carbon released by Foreign producers in supplying Home's imports, C_e^{HF}.

Table 1: Carbon Accounting Matrix

	Home	Foreign	Total
Home	$C_e^{HH} = \int_0^1 e_j(z_j^y)y_j dj$	$C_e^{HF} = \tau^* \int_0^1 e_j^*(z_j^m)m_j dj$	$C_e = C_e^{HH} + C_e^{HF}$
Foreign	$C_e^{FH} = \tau \int_0^1 e_j(z_j^x)x_j dj$	$C_e^{FF} = \int_0^1 e_j^*(z_j^*)y_j^* dj$	$C_e^* = C_e^{FH} + C_e^{FF}$
Total	$G_e = C_e^{HH} + C_e^{FH}$	$G_e^* = C_e^{HF} + C_e^{FF}$	$G_e^W = C_e^W = Q_e^W$.

3 The Planning Problem

Home's planning problem is to allocate the resources that it controls to maximize its welfare, subject to three constraints: (i) its use of labor in the three sectors of the economy cannot exceed its supply of labor; (ii) the global use of energy in manufacturing cannot exceed global extraction of energy; and (iii) its policies cannot make Foreign worse off. To meet the Foreign welfare constraint, the planner can adjust transfers of services from Home to Foreign, subject to $C_s + C_s^* = Q_s + Q_s^*$. The planner is not constrained by trade balance. Consumption, production, and energy extraction in Foreign are dictated by market prices. We consider outcomes in Foreign and set out the constraints below before stating the planning problem.

3.1 Foreign

Energy extractors in Foreign can sell energy at price p_e and can hire labor at wage $w^* = 1$. They tap all deposits with a labor requirement below p_e:

$$Q_e^* = E^*(p_e), \tag{6}$$

or if $p_e < \underline{a}^*$ then $Q_e^* = 0$. Goods producers can purchase energy at price p_e and can hire labor at wage $w^* = 1$. Their cost-minimizing energy intensity is $z^* = (1-\alpha)/(\alpha p_e)$. They supply good j at a price that covers their unit cost:

$$p_j^* = l_j^*(z^*) + p_e e_j^*(z^*) = a_j^* p_e^{1-\alpha}. \tag{7}$$

Consumers in Foreign have the option to purchase any good j from domestic producers at price p_j^*. This price puts a lower bound on their consumption: $c_j^* \geq \eta^* p_j^{*-\sigma^*}$.

3.2 Constraints

3.2.1 Home's Labor Constraint

Combining (1) and (2), the labor L_e required to extract a quantity of energy Q_e is:

$$L_e = \int_0^{E^{-1}(Q_e)} a \, dE(a). \tag{8}$$

The labor L_g required in goods production is:

$$L_g = \int_0^1 \left(l_j(z_j^y) y_j + \tau l_j(z_j^x) x_j \right) dj.$$

Taking account of labor to provide services, $L_s = Q_s$, Home's labor constraint is:

$$L_e + L_g + L_s = L. \tag{9}$$

3.2.2 Global Energy Constraint

The global constraint on use of energy is:

$$G_e + G_e^* \leq Q_e + Q_e^*, \tag{10}$$

where Q_e is chosen by the planner and Q_e^* is given by (6). Expressions for G_e and G_e^*, the quantity of energy used in production, are in the last row of Table 1.

3.2.3 Foreign Welfare Constraint

We require that Home's unilateral policy not reduce welfare in Foreign:

$$C_s^* + (\eta^*)^{1/\sigma^*} \frac{(C_g^*)^{1-1/\sigma^*} - 1}{1 - 1/\sigma^*} - \varphi^* \left(Q_e + Q_e^*\right) = U_0^*, \tag{11}$$

where U_0^* is Foreign welfare in the absence of policy (or business as usual). In evaluating (11) below, we will employ the Foreign analog of (9): $L_e^* + L_g^* + L_s^* = L^*$.

3.3 The Planner's Lagrangian

The planner's objective is to maximize Home welfare:

$$U = C_s + \eta^{1/\sigma} \frac{C_g^{1-1/\sigma} - 1}{1 - 1/\sigma} - \varphi \left(Q_e + Q_e^*\right),$$

subject to the three constraints above: (9), (10), and (11). Substituting in the labor constraint (9) and the Foreign welfare constraint (11), in place of

C_s, the objective becomes global welfare:[17]

$$U = \eta^{1/\sigma}\frac{C_g^{1-1/\sigma}-1}{1-1/\sigma} + (\eta^*)^{1/\sigma^*}\frac{(C_g^*)^{1-1/\sigma^*}-1}{1-1/\sigma^*} - \varphi^W\left(Q_e + Q_e^*\right)$$
$$+ L + L^* - L_e - L_e^* - L_g - L_g^* - U_0^*,$$

where $\varphi^W = \varphi + \varphi^*$ is the global marginal harm from emissions.

To derive the Lagrangian, we apply a Lagrange multiplier λ_e to the energy constraint. The resulting Lagrangian, after dropping constants such as L, L^*, and U_0^*, becomes:

$$\mathcal{L} = \frac{\eta^{1/\sigma}}{1-1/\sigma}\int_0^1 (y_j + m_j)^{1-1/\sigma}\, dj + \frac{(\eta^*)^{1/\sigma^*}}{1-1/\sigma^*}\int_0^1 \left(y_j^* + x_j\right)^{1-1/\sigma^*} dj - \varphi^W Q_e^W$$
$$-L_e^W - \int_0^1 \left(l_j(z_j^y)y_j + \tau l_j(z_j^x)x_j + l_j^*(z^*)y_j^* + \tau^* l_j^*(z_j^m)m_j\right) dj \qquad (12)$$
$$-\lambda_e\left(\int_0^1 \left(e_j(z_j^y)y_j + \tau e_j(z_j^x)x_j + e_j^*(z^*)y_j^* + \tau^* e_j^*(z_j^m)m_j\right) dj - Q_e^W\right).$$

The terms in the Lagrangian are, line by line: (i) utility from goods consumption in Home and Foreign less harm from emissions , (ii) the opportunity cost (in terms of lost consumption of services) from labor employed in energy extraction and goods production, and (iii) the global energy constraint.

Because Home's objective is global welfare, the Lagrangian encompasses a number of different cases which are determined by the resources that Home is assumed to control. In our core planning problem, to derive the *unilateral* optimum, Home can choose the quantities of each good it consumes and each good that it exports, $\{y_j\}$, $\{x_j\}$, $\{m_j\}$, their energy intensities, $\{z_j^y\}$, $\{z_j^x\}$, $\{z_j^m\}$, its level of extraction Q_e, and the price of energy, p_e. To derive the

[17] Accounting for labor constraints, the supply of global services is:

$$C_s + C_s^* = L + L^* - L_e - L_e^* - L_g - L_g^*.$$

Substituting in the Foreign welfare constraint, in place of Foreign's consumption of services, yields an expression for Home's consumption of services:

$$C_s = L + L^* - L_e - L_e^* - L_g - L_g^* + (\eta^*)^{1/\sigma^*}\frac{(C_g^*)^{1-1/\sigma^*}-1}{1-1/\sigma^*} - \varphi^*\left(Q_e + Q_e^*\right) - U_0^*.$$

Substituting into Home's welfare yields the new expression for the planner's objective.

global optimum, Home (or equivalently the global planner in this case) can also choose $\{y_j^*\}$, $\{z_j^*\}$, and Q_e^*.[18] Restricting Home's choices to a narrow set of variables allows us to derive simpler or restricted policies to Home's unilateral optimum (which we explore in Part 5 and in our simulations).

We solve the maximization problem, starting with what CDVW call the *inner problem*, involving optimality conditions for an individual good given values for Q_e, λ_e, and p_e. We then evaluate the optimality conditions for Q_e and p_e in what they call the *outer problem.* The Lagrange multiplier λ_e will be solved to clear the energy market.

To give the inner problem a natural interpretation, we assume for now that if $\varphi^W > 0$, then $\lambda_e > p_e$. We will verify that condition later using results from the outer problem. If $\varphi^W = 0$, then $\lambda_e = p_e$, which allows our solution to Home's planning problem to immediately give us the business as usual or BAU outcome for comparison.

3.4 Inner Problem

The inner problem is to maximize a Lagrangian for any arbitrary good j:

$$\begin{aligned}\mathcal{L}_j = &\frac{\eta^{1/\sigma}}{1-1/\sigma}\left(y_j+m_j\right)^{1-1/\sigma} + \frac{(\eta^*)^{1/\sigma^*}}{1-1/\sigma^*}\left(y_j^*+x_j\right)^{1-1/\sigma^*} \\ &- \left(l_j(z_j^y)y_j + \tau l_j(z_j^x)x_j + l_j^*(z^*)y_j^* + \tau l_j^*(z_j^m)m_j\right) \\ &- \lambda_e\left(e_j(z_j^y)y_j + \tau e_j(z_j^x)x_j + e_j^*(z^*)y_j^* + \tau^* e_j^*(z_j^m)m_j\right).\end{aligned}$$

We consider, in turn: (i) optimal energy intensities, z_j^y, z_j^m, and z_j^x; (ii) optimal quantities for Home consumers, y_j and m_j; and (iii) optimal quantities for Foreign consumers, x_j.

3.4.1 Energy Intensity

The optimal value for energy intensities z_j^y and z_j^x solves $\min_z\{l_j(z)+\lambda_e e_j(z)\}$ while for z_j^m it solves $\min_z\left\{l_j^*(z)+\lambda_e e_j^*(z)\right\}$. Using (5) it is apparent that in all three cases, the optimal energy intensity is:

$$z_j^y = z_j^m = z_j^x = \frac{1-\alpha}{\alpha\lambda_e} = z.$$

[18]In the global planner's problem p_e is redundant. Appendix A provides a step-by-step solution.

The planner chooses a common energy intensity z for the production of any good consumed in Home (whether produced in Home or Foreign) and for all production in Home (whether serving consumers in Home or Foreign). In the BAU scenario we get $z_{BAU} = z^*$, so that all goods, including those produced and consumed in Foreign, are produced at a common energy intensity.

For any good produced in Home for domestic consumption the energy requirement is:

$$e_j(z) = (1-\alpha)a_j\lambda_e^{-\alpha}$$

while the overall shadow cost is

$$l_j(z) + \lambda_e e_j(z) = a_j\lambda_e^{1-\alpha}.$$

If good j is exported from Home, the shadow cost is $\tau a_j\lambda_e^{1-\alpha}$, while if it is imported by Home, the shadow cost is

$$\tau^*\left(l_j^*(z) + \lambda_e e_j^*(z)\right) = \tau^* a_j^*\lambda_e^{1-\alpha}.$$

3.4.2 Goods for Home Consumers

The first order conditions for y_j and m_j, after substituting in results for shadow costs, can be written as:

$$\eta^{1/\sigma}(y_j + m_j)^{-1/\sigma} - a_j\lambda_e^{1-\alpha} \leq 0,$$

with equality if $y_j > 0$ and

$$\eta^{1/\sigma}(y_j + m_j)^{-1/\sigma} - \tau^* a_j^*\lambda_e^{1-\alpha} \leq 0,$$

with equality if $m_j > 0$. The implications of these two FOC's are easy to distill by defining the good $\bar{j}_m$ for which they both hold with equality. Applying (4), this cutoff good satisfies:

$$F(\bar{j}_m) = \frac{1}{\tau^*}. \tag{13}$$

For goods $j < \bar{j}_m$, Home has a comparative advantage, the second FOC holds with a strict inequality so that $m_j = 0$, and the first holds with equality to determine y_j. For goods $j > \bar{j}_m$, Foreign has a comparative advantage, the first FOC holds with a strict inequality so that $y_j = 0$, and the second holds with equality to determine m_j. (We can ignore the outcome for the measure-zero threshold good $j = \bar{j}_m$.) The results are summarized in the first row of Table 2.

3.4.3 Goods for Foreign Consumers

Foreign's marginal utility from consumption of good j is bounded above by p_j^*, the cost (7) at which it can supply the good to itself. Whether or not that upper bound binds makes two cases to consider in determining the optimal x_j. Case I pertains to goods j for which Foreign's marginal utility remains strictly below p_j^*. In this case we can set $y_j^* = 0$ so that Foreign consumption is $c_j^* = x_j$. Case II pertains to goods j for which Foreign's marginal utility equals p_j^*. In this case c_j^* is invariant to a decline in x_j as it will be exactly offset by a rise in y_j^* that keeps marginal utility equal to p_j^*.

Consider a good j in Case I. The first order condition for x_j, after substituting in the result for Home's shadow cost of serving the export market, can be written as:

$$(\eta^*)^{1/\sigma^*} x_j^{-1/\sigma^*} - \tau a_j \lambda_e^{1-\alpha} = 0.$$

The first term, which is Foreign's marginal utility, is thus equated to the shadow cost, $\tau a_j \lambda_e^{1-\alpha}$. This cost is strictly below p_j^* for any good $j < j_0$, where j_0 satisfies:

$$F(j_0) = \tau \left(\frac{\lambda_e}{p_e} \right)^{1-\alpha}. \tag{14}$$

Case I applies to $j \in [0, j_0)$.

Now consider a good j in Case II, so that $j \geq j_0$. Foreign's marginal utility no longer depends on x_j, since c_j^* is fixed. But, the resource savings in Foreign from a change in x_j does enter, since $y_j^* = c_j^* - x_j$. After substituting in the relevant shadow values, the derivative of the Lagrangian is:

$$\frac{\partial \mathcal{L}_j}{\partial x_j} = -\tau a_j \lambda_e^{1-\alpha} + l_j^*(z^*) + \lambda_e e_j^*(z^*). \tag{15}$$

The last two terms represent the value that the planner places on the labor (valued at 1) and energy (valued at λ_e) that Foreign would have used to produce an additional unit of good j for itself. This derivative is predicated on $y_j^* > 0$, but otherwise doesn't depend on x_j. The good $j = \bar{j}_x$ at which (15) is zero satisfies:

$$F(\bar{j}_x) = \tau \frac{\left(\frac{\lambda_e}{p_e} \right)^{1-\alpha}}{\alpha + (1-\alpha)\frac{\lambda_e}{p_e}}. \tag{16}$$

Having posited $\lambda_e > p_e$ it follows that $\bar{j}_x > j_0$. For any good $j > \bar{j}_x$, Foreign has a strong comparative advantage and $x_j = 0$ since the value that the planner places on the resources saved in Foreign doesn't offset the shadow cost of Home producing the good for export. (We can ignore the outcome for the measure-zero threshold good $j = \bar{j}_x$.)

For any good $j \in (j_0, \bar{j}_x)$ Home's comparative advantage is stronger so that (15) is strictly positive. Exports are pushed to the non-differentiable limit at which y_j^* is driven to zero. The quantity exported equates Foreign's marginal utility to p_j^*:

$$(\eta^*)^{1/\sigma^*} x_j^{-1/\sigma^*} - a_j^* p_e^{1-\alpha} = 0.$$

Case II applies to goods $j \in (j_0, \bar{j}_x)$. The results are summarized in the second row of Table 2.

The BAU case is much simpler, and can be seen by setting $\lambda_e = p_e$ in (16), giving us an export threshold $\bar{j}_{x,BAU} = j_{0,BAU}$ satisfying $F(\bar{j}_{x,BAU}) = \tau$. The import threshold $\bar{j}_m$ satisfies (13), as in the unilateral optimum.

Table 2 displays the terms for each of the four quantities of good j. As in Table 1, the rows indicate the location of consumption while the columns indicate the location of production. These terms are as expected except for Home exports, x_j, for goods $j \in (j_0, \bar{j}_x)$: (i) exports of such goods reflect the price of energy p_e in Foreign rather than Home's shadow price λ_e, (ii) although produced in Home, they reflect Foreign productivity a_j^* rather than Home productivity a_j, and (iii) they do not reflect the iceberg costs of export τ. That is, $x_j \neq \eta^* \left(\tau a_j \lambda_e^{1-\alpha}\right)^{-\sigma}$ as is the case when $j \leq j_0$. The reason is that for goods $j \in (j_0, \bar{j}_x)$ Home crowds out Foreign production, in order to produce these goods with lower energy intensity, but its comparative advantage in these goods is not strong enough to justify exporting enough to push Foreign marginal utility below p_j^*.

3.5 Outer Problem

We now turn to the optimality conditions for Q_e and p_e, rewriting the Lagrangian in terms of aggregate magnitudes:

$$\begin{aligned} \mathcal{L} = & \frac{\eta^{1/\sigma}}{1 - 1/\sigma} C_g^{1-1/\sigma} + \frac{(\eta^*)^{1/\sigma^*}}{1 - 1/\sigma^*} (C_g^*)^{1-1/\sigma^*} - \varphi^W (Q_e + Q_e^*) \\ & - L_e - L_e^* - L_g - L_g^* - \lambda_e (G_e + G_e^* - Q_e - Q_e^*) . \end{aligned} \tag{17}$$

Table 2: Production and Distribution of a Good

	Home		Foreign	
Home	$y_j = \eta \left(a_j \lambda_e^{1-\alpha}\right)^{-\sigma}$	$j < \bar{j}_m$	$m_j = \eta \left(\tau^* a_j^* \lambda_e^{1-\alpha}\right)^{-\sigma}$	$j > \bar{j}_m$
Foreign	$x_j = \begin{cases} \eta^* \left(\tau a_j \lambda_e^{1-\alpha}\right)^{-\sigma^*} \\ \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*} \end{cases}$	$j < j_0$ $j_0 < j < \bar{j}_x$	$y_j^* = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*}$	$j > \bar{j}_x$

3.5.1 Energy Extraction

The first order condition with respect to Q_e is:

$$\frac{\partial \mathcal{L}}{\partial Q_e} = -\varphi^W - \frac{\partial L_e}{\partial Q_e} + \lambda_e \leq 0,$$

with equality if $Q_e > 0$. The extra labor in Home to extract a bit more energy is the labor requirement for the marginal energy deposit, $E^{-1}(Q_e)$.[19] Applying this inverse, the first order condition simplifies to:

$$Q_e = E\left(\lambda_e - \varphi^W\right), \tag{18}$$

for $\lambda_e - \varphi^W \geq \underline{a}$ and $Q_e = 0$ otherwise. The BAU scenario has p_e in place of $\lambda_e - \varphi^W$, as with Foreign extraction (6).

[19] Integrating (8) by parts, it becomes:

$$L_e = E^{-1}(Q_e)Q_e - \int_0^{E^{-1}(Q_e)} E(a)da.$$

Differentiating it in this form:

$$\frac{\partial L_e}{\partial Q_e} = E^{-1}(Q_e) + Q_e \frac{\partial E^{-1}}{\partial Q_e} - E(E^{-1}(Q_e)) \frac{\partial E^{-1}}{\partial Q_e} = E^{-1}(Q_e).$$

3.5.2 Energy Price

The first order condition with respect to p_e can be written as:

$$\left(\frac{\eta^*}{C_g^*}\right)^{1/\sigma^*}\frac{\partial C_g^*}{\partial p_e} - \varphi^W\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial L_e^*}{\partial p_e} - \frac{\partial L_g}{\partial p_e} - \frac{\partial L_g^*}{\partial p_e} = \lambda_e\left(\frac{\partial G_e}{\partial p_e} + \frac{\partial G_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e}\right).$$

To make sense of this condition requires computing the partial derivatives (with respect to the energy price) of C_g^*, Q_e^*, L_e^*, L_g, L_g^*, G_e, and G_e^*, each evaluated at the optimal unilateral policy itself.

Foreign energy extraction depends directly on the energy price, via (6), so that $\partial Q_e^*/\partial p_e = \partial E^*(p_e)/\partial p_e$. The response of Foreign employment in the energy sector is $\partial L_e^*/\partial p_e = p_e \partial Q_e^*/\partial p_e$.

Dependence on the energy price is more subtle for the other aggregates as pieces of them have already been chosen by the planner in the inner problem. For example, energy use by Foreign producers:

$$G_e^* = \int_{\bar{j}_x}^1 e_j^*(z^*)y_j^* dj + \tau^* \int_{\bar{j}_m}^1 e_j^*(z)m_j dj,$$

depends on the energy price only through the first integral, C_e^{FF} . The partial derivative we seek is therefore:

$$\frac{\partial G_e^*}{\partial p_e} = -\epsilon_D^* \frac{C_e^{FF}}{p_e} < 0.$$

That is, a change in the energy price affects Foreign's use of energy only through its domestic consumption C_e^{FF} and not through its exports of goods to Home C_e^{HF}. Home has chosen and optimized the determinants of C_e^{HF} ($\bar{j}_m$, m_j, and $z^m = z$).

In Appendix B we compute all the partial derivatives and substitute them into the first order condition above to get:

$$\left((\lambda_e - \varphi^W) - p_e\right)\frac{\partial Q_e^*}{\partial p_e} = (\lambda_e - p_e)\frac{\partial G_e^*}{\partial p_e} + \int_{j_0}^{\bar{j}_x}\left(\tau a_j \lambda_e^{1-\alpha} - p_j^*\right)\frac{\partial x_j}{\partial p_e}dj. \quad (19)$$

This optimality condition balances Foreign supply and demand responses to a change in p_e, weighting them by the three wedges introduced in the optimal unilateral policy: (i) the wedge between the marginal valuation of extraction in Home and the price obtained by extractors in Foreign, (ii) the wedge between the shadow cost of energy to producers in Home and the actual cost of energy to Foreign producers, and (iii) the wedge

$$s_j = \tau a_j \lambda_e^{1-\alpha} - p_j^*,$$

for each good $j \in (j_0, \bar{j}_x)$, between Home's shadow cost of supplying exports of j and the marginal utility to consumers in Foreign.[20]

We get a more compact expression by introducing Home's implicit export subsidy:

$$S = \int_{j_0}^{\bar{j}_x} s_j x_j dj.$$

We can rewrite (19) as:

$$\lambda_e - p_e = \frac{\varphi^W \epsilon_S^* Q_e^* - \sigma^*(1-\alpha)S}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}}. \tag{20}$$

where ϵ_S^* is the Foreign elasticity of energy supply:

$$\epsilon_S^* = \frac{\partial E^*(p_e)}{\partial p_e} \frac{p_e}{E^*}. \tag{21}$$

and ϵ_D^* is Foreign's elasticity of demand for embodied energy:

$$\epsilon_D^* = \alpha + (1-\alpha)\sigma^*. \tag{22}$$

The value of ϵ_D^* captures both the elasticity of the unit energy requirement in production, α, and the elasticity of the price of the good with respect to the energy price, $1-\alpha$, multiplied by σ^*.

If Foreign extracts a lot of energy and the quantity is sensitive to the energy price, i.e. if $\epsilon_S^* Q_e^*$ is large relative to $\epsilon_D^* C_e^{FF}$, then the planner wants to keep Foreign's energy price low by raising λ_e. If the planner's implicit subsidy of exports is high, it wants to keep the price of energy high by lowering λ_e.

[20] The last term appears qualitatively different from the first two since it involves the price response to consumption of good j rather than to the implicit demand for the energy embodied in it, which we can denote by $E_j^x = \tau e_j(z) x_j$. This apparent difference goes away if we rewrite the integral as:

$$\int_{j_0}^{\bar{j}_x} \frac{s_j}{\tau e_j(z)} \frac{\partial E_j^x}{\partial p_e} dj = \frac{\lambda_e}{1-\alpha} \int_{j_0}^{\bar{j}_x} \left(1 - \frac{F(j)}{F(j_0)}\right) \frac{\partial E_j^x}{\partial p_e} dj.$$

The term $F(j)/F(j_0)$, which is below 1 for $j > j_0$, is Home's relative efficiency in producing good j divided by the value that would equate Home's shadow cost of delivering exports of j with Foreign's cost of supplying itself.

3.6 Properties of the Solution

We can now compute the optimal policy in principle: (i) the inner problem gives G_e and G_e^* in terms of p_e and λ_e, (ii) equations (6) and (18) give Q_e^* and Q_e as functions of p_e and λ_e, and (iii) equation (19) and the global energy constraint (10), which binds, jointly nail down p_e and λ_e. The expressions above provide insight into the properties of this solution. These insights build on the result that $\lambda_e > p_e$ if $\varphi^W > 0$, which we asserted above and can now prove. In the discussion that follows, unless otherwise noted, we take $\varphi^W > 0$ (if $\varphi^W = 0$ then $\lambda_e = p_e$).

3.6.1 Bounds on $\lambda_e - p_e$

We start by establishing a lower bound on $\lambda_e - p_e$ by decomposing the term S in (20). It consists of the implicit subsidy s_j for each good $j \in (j_0, \bar{j}_x)$. Adding and subtracting $\lambda_e e_j^*(z^*)$ from that implicit subsidy:

$$s_j = (\lambda_e - p_e)e_j^*(z^*) - (l_j^*(z^*) + \lambda_e e_j^*(z^*) - \tau a_j \lambda_e^{1-\alpha}).$$

The first term is the difference in the value that the planner and Foreign place on the energy used by Foreign to produce a unit of good j. The remaining term, $\pi_j = l_j^*(z^*) + \lambda_e e_j^*(z^*) - \tau a_j \lambda_e^{1-\alpha}$ is the planner's value of the global resources saved when a unit of good j is produced in Home and exported rather than being produced in Foreign. From (15) note that π_j is also the derivative of the inner problem with respect to x_j, so is strictly positive for $j < \bar{j}_x$ and zero at $j = \bar{j}_x$. Substituting this new expression for s_j into the overall implicit subsidy S and rearranging, we can rewrite (20) as:

$$\lambda_e - p_e = \frac{\varphi^W \epsilon_S^* Q_e^* + \sigma^*(1-\alpha)\int_{j_0}^{\bar{j}_x} \pi_j x_j dj}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF} + \int_{j_0}^{\bar{j}_x} e_j^*(z^*) x_j dj}.$$

Both the numerator and denominator are strictly positive, establishing the result that $\lambda_e - p_e > 0$.

Having shown that $\lambda_e > p_e$, it follows that $j_0 < \bar{j}_x$ and hence $S > 0$. We get an upper bound on λ_e by using (20) to write:

$$\varphi^W - (\lambda_e - p_e) = \frac{\epsilon_D^* C_e^{FF}}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}}\varphi^W + \frac{\sigma^*(1-\alpha)}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}} S.$$

The right-hand side is strictly positive, which implies $\lambda_e - p_e < \varphi^W$.

The planner picks a value of $\lambda_e - p_e$ within this range of $(0, \varphi^W)$. It will approach the upper bound if $\epsilon_S^* Q_e^* / (\epsilon_D^* C_e^{FF})$ is large. In this case the planner chooses a low energy price to keep Foreign extraction in check. It will approach the lower bound if $\epsilon_S^* Q_e^* / \epsilon_D^* C_e^{FF}$ is low. In this case the planner chooses a high price to reduce Foreign demand.

3.6.2 Basic Properties

We can now enumerate the basic properties of the optimal policy, using the BAU outcome (obtained by setting $\varphi^W = 0$) as a point of comparison:

1. Energy intensity is lower than under BAU for goods either produced or consumed in Home, $z < z_{BAU} = z^*$.

2. Home reduces consumption of all goods, and Foreign reduces consumption of goods $j < j_0$, relative to BAU.

3. While the import threshold, $\bar{j}_m$, is the same as for BAU, the export threshold expands, $\bar{j}_x > \bar{j}_{x,BAU}$.[21]

4. Home implicitly subsidizes exports of goods $j > j_0$, where $j_0 < \bar{j}_{x,BAU}$.

5. Home energy extraction is lower than under BAU, as can be seen from (18), taking account of the upper bound on λ_e.

3.6.3 Crosshauling

The fact that the import threshold remains unchanged from BAU while the export threshold rises creates the possibility for crosshauling. Under the optimal policy there may be a set of goods that Home simultaneously imports and exports. Such a set of goods always exists with no trade costs, $\tau = \tau^* = 1$, since then $F(\bar{j}_m) = 1$ while $F(\bar{j}_x) < 1$ (as shown in footnote 24) so that $\bar{j}_x > \bar{j}_m$.

[21]Evaluating (16) at $\lambda_e/p_e = 1$ gives $F(\bar{j}_x) = \tau$, hence $\bar{j}_x = \bar{j}_{x,BAU}$. Differentiating (16) with respect to λ_e/p_e:

$$\frac{\partial F(\bar{j}_x)}{\partial(\lambda_e/p_e)} = \alpha(1-\alpha)\frac{F(\bar{j}_x)^2}{F(j_0)}\left(\frac{1}{\lambda_e/p_e} - 1\right),$$

which is negative for $\lambda_e/p_e > 1$. Hence $\bar{j}_x$ exceeds $\bar{j}_{x,BAU}$ for $\lambda_e > p_e$.

The economic rationale for crosshauling is that Home controls energy intensity not only for all production in Home, but also for production in Foreign that Home imports. In contrast, goods produced in Foreign, for consumption there, escape Home's carbon policy.

Increased trade gives Home more control over the use of energy, helping it to lower global emissions. In particular, if trade costs are low enough, Home is willing to supply a range of goods to Foreign at a price below the shadow value of those goods to Home consumers, which is the rationale for the implicit export subsidy s_j.

To illustrate, continue to assume no trade costs and consider good $j = \bar{j}_m$ so that $a_{\bar{j}_m} = a^*_{\bar{j}_m} = a$. Home is indifferent between importing this good or producing it for itself. If λ_e were equal to p_e Home would also be indifferent between exporting this good or having Foreign produce it for itself. Since $\lambda_e > p_e$ global energy use is reduced if the good is produced in Home and exported to Foreign. The energy requirement is $(1-\alpha)\, a\lambda_e^{-\alpha}$ which is less than if Foreign produced for itself, with energy requirement $(1-\alpha)\, ap_e^{-\alpha}$ per unit produced. This energy savings is not a sufficient condition for Home to export the good, however, since that depends on the total net resource savings, including labor. The optimality of exporting the good is because, combining labor and energy inputs, and weighting energy by its shadow value of λ_e, the net resource savings is: $\alpha a p_e^{1-\alpha} + \lambda_e(1-\alpha)ap_e^{-\alpha} - a\lambda_e^{1-\alpha} > 0$.[22]

Trade costs mute this effect. With high enough trade costs, $F(\bar{j}_x) > F(\bar{j}_m)$ so that $\bar{j}_x < \bar{j}_m$. The inherent inefficiency of crosshauling overcomes its advantage in reducing global emissions. Yet, even in this case, the optimal policy broadens the range of goods that Home exports.

4 Optimal Taxes and Subsidies

We now describe a set of taxes and subsidies that deliver the optimal outcomes in a competitive equilibrium. In shifting from a planning problem to a market economy, recall that services are the numéraire and the unit labor requirement for services pins the wage to 1 in both countries. We treat p_e as the *global* energy price, the base to which we apply carbon taxes.

[22]The inequality is due to the fact that the net resource savings is 0 at $\lambda_e = p_e$ but increases with λ_e for $\lambda_e > p_e$. More generally, the net resource savings is simply π_j evaluated at $j = \bar{j}_m$. From our results above we know that $\pi_j > 0$ for $j \in (j_0, \bar{j}_x)$ and, with no trade costs, $\bar{j}_m \in (j_0, \bar{j}_x)$.

The taxes and subsidies we introduce into this competitive equilibrium must generate the three wedges that appear in the optimal policy:

1. Energy extractors in Home face a wedge $(\lambda_e - \varphi^W) - p_e$ between their marginal returns and those of their Foreign counterparts.

2. Energy users in Home face a wedge $\lambda_e - p_e$ between their cost of energy and the cost to Foreign users.

3. Exporters face a wedge $s_j = \tau a_j \lambda_e^{1-\alpha} - p_j^*$ between the cost of producing and shipping any good $j \in (j_0, \bar{j}_x)$ and the marginal utility experienced by Foreign consumers.

While these wedges are unique, the policies that deliver them are not.

4.1 A Simple Implementation

We focus on a policy that is easy to describe, with three elements of intervention:

1. Impose a nominal tax on Home's energy extraction, t_e, set at the Pigouvian rate:
$$t_e = \varphi^W.$$

2. Impose a border adjustment, t_b, on Home's imports or exports of energy and on the energy content of Home's imports of goods, set to the difference between λ_e and p_e:
$$t_b = \frac{\varphi^W \epsilon_S^* Q_e^* - \sigma^*(1-\alpha)S}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}}. \tag{23}$$

3. Provide an export subsidy s_j, per unit exported of any good $j \in (j_0, \bar{j}_x)$ of:
$$s_j = a_j(p_e + t_b)^{1-\alpha} - p_j^*.$$

The border adjustment is partial in the sense that it is at a lower rate than the underlying extraction tax and it is one-way because it applies only to imports and exports of energy and imports of goods, but not to the export of goods.[23]

[23] Our description of the optimal policy uses nominal taxes and border adjustments. An equivalent way to describe the taxes is to use effective taxes. In particular, a nominal tax

4.2 After-Tax Prices

To eliminate ambiguity about how this policy would work, we list the net prices faced by the different agents in the global economy:

1. The global price of energy, prior to any taxes, is p_e. This price is paid by users of energy in Foreign and is received by energy extractors in Foreign.

2. If energy is imported by Home, it is subject to a border adjustment t_b, raising the price of energy for users in Home to $p_e + t_b$.

3. Energy extractors in Home sell energy domestically at price $p_e + t_b$, but their net after paying the extraction tax is $p_e + t_b - t_e$. Exporting brings them a partial rebate of t_b on the extraction tax paid leaving the price received, net of the tax and partial rebate, the same as if they sell domestically.

4. Goods $j < \bar{j}_m$ are produced in Home, using energy costing $p_e + t_b$, so that local consumers pay $p_j = a_j(p_e + t_b)^{1-\alpha}$.

5. Goods $j > \bar{j}_m$ are imported by Home. Foreign produces them with energy intensity z in anticipation of the border adjustment levied on their energy content. Their cost of production (including delivery to Home) is $\tau^* l_j^*(z) + p_e \tau^* e_j^*(z)$. (Note that the energy they use is not taxed directly.) When the border adjustment, $t_b \tau^* e_j^*(z)$, is added on, however, the price to consumers in Home becomes, $p_j^m = \tau^* a_j^*(p_e + t_b)^{1-\alpha}$.

6. Goods $j < j_0$ are produced in Home and exported. The producers use energy costing $p_e + t_b$ with no adjustment when the goods are exported. The price in Foreign, including the trade cost, is $p_j^x = \tau a_j(p_e + t_b)^{1-\alpha}$.

7. Goods $j \in (j_0, \bar{j}_x)$ are also produced in Home and exported. The producers use energy priced at $p_e + t_b$, with no relief from the energy tax when the goods are exported. They sell at price p_j^* in Foreign, but get a subsidy from Home of s_j per unit so that they are able to cover their cost: $p_j^* + s_j = \tau a_j(p_e + t_b)^{1-\alpha}$.

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on extraction of t_e and a border adjustment on energy of t_b can equivalently be described as an effective tax on extraction of $\tilde{t}_e = t_e - t_b$ and an effective tax on production of $\tilde{t}_p = t_b$. We use nominal taxes because the optimal policy is simpler to express in those terms.

8. Goods $j > \bar{j}_x$ are produced in Foreign, using energy at price p_e. They are sold to local consumers at price $p_j^* = a_j^* p_e^{1-\alpha}$.

4.3 Economic Principles

We can understand Home's optimal policy in terms of how its policy affects activities in Foreign that are outside of its direct control. In particular, Home cannot directly control Foreign's extraction, and it cannot directly control Foreign's production of goods that are consumed in Foreign. The latter has two components: the energy intensity of the production (the intensive margin) and the set of goods that Foreign producers sell to Foreign consumers (the extensive margin). These three margins—Foreign extraction, the energy intensity of Foreign production, and the set of goods Foreign producers sell to Foreign consumers—can be thought of as three different sources of leakage. Home sets its combination of an extraction tax, a border adjustment, and an export subsidy to indirectly affect these margins, in effect, controlling these three sources of leakage.

To understand these interactions, consider a pure extraction tax in Home (i.e., an extraction tax with no border adjustment or export subsidy). The tax increases the price of energy, resulting in an increase in Foreign extraction, or "extraction leakage." If Foreign's price elasticity of energy supply is high, extraction leakage is high, resulting in costs to Home that go up with φ^W.

Border adjustments on energy moderate this effect. Increasing the border adjustment lowers the price of energy, thereby reducing extraction leakage. Lowering p_e, however, introduces distortions on the production and consumption side. As p_e goes down, the set of goods produced in Foreign increases, and Foreign's energy use in production of those goods goes up. The set of goods produced in Foreign roughly corresponds to traditional (production) leakage, while the energy intensity of those goods is sometimes called the "fuel price effect."[24]

The optimal border adjustment balances extraction leakage and the fuel price effect (and as we discuss below, production leakage). Its level depends on

[24]These terms, however, are not clearly distinguished in the literature, and our use of them is only suggestive. The fuel price effect appears to refer to any change in Foreign production or consumption due to a reduction in p_e. If true, then traditional production leakage is limited to shifts in import or export margins holding p_e fixed. Our usage does not precisely correspond to these definitions because our expressions all use the equilibrium value of p_e.

how Foreign supply and demand for energy respond to a change in the price of energy. If Foreign's elasticity of supply is high relative to demand, Home will want a high border adjustment, possibly approaching φ^W. Conversely, if Foreign energy supply is inelastic relative to Foreign energy demand, the border adjustment will be low, possibly approaching zero. In the extreme of a "vertical" Foreign supply curve (so that $\epsilon_S^* = 0$, at least locally) the planner may be able to achieve the global optimum by cutting back on its own extraction.[25] The principle of combining these two tax instruments, t_e and t_b, is at the heart of the seminal paper of Markusen (1975).[26]

Home controls production leakage through a combination of a border adjustment on imports and a goods-specific subsidy for exports. As discussed immediately above, the border tax on imports means that imports face the same effective energy price as goods produced in Home. As a result, the border tax leaves the extensive margin for imports the same as without tax and causes the energy intensity of imports to be the same as that of goods produced in Home.

Home could control the export margin in a parallel fashion, by rebating taxes on export. Doing so would leave the export margin the same as it would be without tax. This policy, however, would remove the incentive for exporters to lower their energy intensity. Rather than removing the tax on export, therefore, Home offers good specific subsidies. Because these subsidies do not depend on energy usage, they retain incentives for exporters to produce goods with low energy intensity.[27]

The subsidy goes beyond merely restoring Home's export margin: it applies to goods for which Home would not be competitive in the absence of any carbon policy. The reason follows the argument above for potential

[25] Following this logic, Harstad (2012) makes a case that the policy maker buy marginal energy deposits from Foreign to create a locally vertical Foreign supply curve. We have ruled out such an international market in Foreign energy deposits in our analysis here.

[26] This connection to Markusen (1975) is disguised by differences in terminology. Our extraction tax is what he refers to as a production tax. Our border adjustment is what he refers to as a trade tax. Furthermore, his taxes are ad valorem while ours are specific. More fundamentally, he imposes trade balance, so that his trade tax incorporates terms-of-trade considerations. Finally, in his model there is no analog of our production sector, which uses energy to produce tradable goods. Hence, his analysis doesn't speak to how the border adjustment applies to the energy embodied in these goods.

[27] This basic logic comes from Fischer and Fox (2012), who point out that rebating carbon tax revenue to producers, in proportion to their production not their tax payments, does not eliminate the incentive for them to use less carbon.

cross-hauling under the optimal policy. The policy is designed to crowd out some of Foreign's energy-intensive goods production for its domestic market. (Note that the same logic does not apply to the import margin because the border tax on imports ensures that all goods consumed in Home are produced with the same (low) energy intensity. The asymmetry between imports and exports arises because Home cannot directly control the energy intensity of goods produced in Foreign that are consumed in Foreign. The export policy seeks to crowd out this activity.

If $\varphi^W = 0$ there are no wedges and the optimal policy sets $t_e = t_b = 0$ and $s_j = 0$ for all j. The outcome is the BAU competitive equilibrium, which arises in the absence of policy even if $\varphi^W > 0$. We take this outcome to be our baseline in the quantitative results that follow.

5 Constrained Optimal Policies

To assess the optimal policy, it is useful to compare it to more conventional policies. We will consider three: (i) a pure extraction tax, (ii) a pure production tax, and (iii) a pure consumption tax. We also consider three hybrids, which are optimal combinations of any two of these three conventional policies. In each case, we return to the planner's Lagrangian (12) to maximize global welfare. Each conventional policy (and hybrid) removes a particular set of variables from the planner's control. Those variables are determined instead as they would be in a competitive equilibrium. Here we simply state the results, relegating the derivations to Appendix C.

5.1 Pure Extraction Tax

To obtain an optimal pure extraction tax we let the planner choose only Q_e and p_e. We solve the Lagrangian for this problem and reinterpret the outcome as a decentralized equilibrium. The optimal extraction tax in this case is:

$$t_e = \varphi^W \frac{\epsilon_D C_e + \epsilon_D^* C_e^*}{\epsilon_S^* Q_e^* + \epsilon_D C_e + \epsilon_D^* C_e^*}. \tag{24}$$

This rate is below the value of $t_e = \varphi^W$ in the optimal policy. How much below turns on the value of $\epsilon_S^* Q_e^*$. If Foreign is a major energy extractor and if its price elasticity of supply is high, then Home will want to choose a lower extraction tax because of concerns raising the extraction tax would stimulate

Foreign extraction. In the optimal policy this concerns would lead to a higher border adjustment, but that option is absent with a pure extraction tax.

5.2 Pure Consumption Tax

For a pure consumption tax, we follow the same procedure except now the planner is constrained to choose: $\left\{z_j^y\right\}$, $\left\{z_j^m\right\}$, $\{y_j\}$, $\{m_j\}$, and p_e, with all other outcomes (including Q_e) determined as in a competitive equilibrium. Expressed as an effective tax (i.e., a tax directly on only consumption), the optimal consumption tax is:

$$\tilde{t}_c = \varphi^W \frac{\epsilon_S Q_e + \epsilon_S^* Q_e^*}{\epsilon_S Q_e + \epsilon_S^* Q_e^* + \epsilon_D^* C_e^*}. \tag{25}$$

Expressed in nominal terms, Home would impose an extraction tax of $t_e = \tilde{t}_c$ and border adjustments on both energy and goods at that same rate, $t_b = \tilde{t}_c$.[28]

As with an extraction tax, the term, multiplying φ^W is less than one, but the amount it is less than one now depends on the relative value of $\epsilon_D^* C_e^*$. If Foreign is a major energy consumer and if its price elasticity of demand is high, then Home will want to choose a lower consumption tax because of concerns that a high consumption tax will generate an increase in Foreign energy consumption. In the optimal policy this concerns would lead to an extraction tax exceeding the border adjustment, but that option is absent with a pure consumption tax.

5.3 Optimal Hybrid

In our hybrid tax, we allow Home to combine extraction taxes and consumption taxes. Solving for the optimal mix is the same as for a pure consumption tax, replacing the competitively determined Q_e with the planner's choice. The optimal tax in this case is:

$$t_e = \varphi^W$$

together with a border adjustment of:

$$t_b = \varphi^W \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*}. \tag{26}$$

[28]Unlike the optimal policy, the border adjustment here applies to Home exports of goods, which means these goods bear no tax.

Since it is a partial border adjustment, unlike the pure consumption tax, the net-of-tax price received by Home's energy extractors is $p_e + t_b - \varphi^W$

The equivalent effective taxes are:

$$\tilde{t}_e = \varphi^W \frac{\epsilon_D^* C_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*}$$

and a consumption tax, $\tilde{t}_c = t_b$ as determined in (26). Expressed this way, these terms have obvious parallels with the pure extraction and consumption tax expressions, (24) and (25). With this hybrid policy, much like in the optimal policy, the planner can optimally trade off the effect of the policy on Foreign extraction and Foreign consumption of energy.[29]

5.4 Pure Production Tax and Hybrids

We find the optimal pure production tax and hybrids involving production taxes numerically.

6 Quantitative Illustration

We now turn to the quantitative implications of the optimal policy. We pursue a strategy, based on Dekle, Eaton, and Kortum (2007), in which we calibrate the BAU competitive equilibrium to data on global carbon flows. We then compute the optimal policy relative to this baseline. Following this approach requires that we find values for only a few additional parameters.

To proceed we first need to specify the comparative advantage curve $F(j)$ and the distributions of energy deposits, $E(a)$ and $E^*(a)$. We then calibrate the baseline BAU competitive equilibrium to data on energy extraction and global carbon flows. From this base, we compute the taxes and subsidies that would shift the model economy to the outcomes dictated by the optimal policy. We also compare the BAU and optimal policies to more conventional policies, in particular to the constrained policies derived in the previous section.

[29] Differences remain, however, since this hybrid policy does not include the same export policy as under the optimal policy. In particular, Home goods exports face no carbon tax under the hybrid policy, nor is the export threshold raised via subsidies.

6.1 Setup

We start by providing some of the details of the simulation procedure (with most of the derivations relegated to the Appendix), and then present our key results.

6.1.1 Functional Forms

To solve the model numerically we employ several convenient functional forms.

Energy Supply We parameterize the upper end of the distribution of energy deposits by treating the supply elasticities, ϵ_S and ϵ_S^*, as constant. In Home, for all $a \geq \underline{a}$:

$$E(a) = Ea^{\epsilon_S} \tag{27}$$

and in Foreign, for all $a \geq \underline{a}^*$:

$$E^*(a) = E^* a^{\epsilon_S^*}, \tag{28}$$

where $E > E(\underline{a})$ and $E^* > E^*(\underline{a}^*)$ are constants. We choose units of energy so that in the BAU baseline the global energy price is 1 and hence baseline extraction is $Q_e = E$ and $Q_e^* = E^*$.[30]

Comparative Advantage We parameterize the efficiency of the goods sector in each country by:

$$a_j = \left(\frac{j}{A}\right)^{1/\theta}, \tag{29}$$

$$a_j^* = \left(\frac{1-j}{A^*}\right)^{1/\theta}, \tag{30}$$

where A and A^* determine absolute advantage in either country, and θ determines (inversely) the scope of comparative advantage. Taking the ratio, we obtain a convenient functional form for Home's comparative advantage curve:

$$F(j) = \frac{a_j^*}{a_j} = \left(\frac{A}{A^*}\frac{1-j}{j}\right)^{1/\theta},$$

[30]This normalization of the baseline energy price requires a corresponding shift of the lower support of the distribution of energy deposits to keep $E(\underline{a})/E$ and $E^*(\underline{a}^*)/E^*$ unchanged.

where $F(j)$ is continuous and strictly decreasing as specified in (4).

This functional form allows us to easily solve for the import and export thresholds in the BAU. In the BAU baseline a country's average spending per good doesn't depend on the source of the good. Since the share of energy in the cost of any good is the same, the baseline value of the import margin is:

$$\bar{j}_m = \frac{C_e^{HH}}{C_e} = \frac{A}{A + (\tau^*)^{-\theta} A^*},$$

while the baseline value of the export margin is:

$$\bar{j}_x = \frac{C_e^{FH}}{C_e^*} = \frac{\tau^{-\theta} A}{\tau^{-\theta} A + A^*}.$$

6.1.2 Calibration of BAU Scenario.

We calibrate the BAU baseline to carbon accounting data for 2015 from the Trade Embodied in CO_2 ($TECO_2$) database made available by the OECD.[31] Units are gigatonnes of CO_2. Energy extraction data for 2015 is from the International Energy Agency World Energy Statistics Database. We use emissions factors to convert units of energy to units of CO_2.

For most of our results, members of the OECD form the taxing region, Home, and the non-OECD countries are Foreign. Table 3 provides our calibration.

Note that by this CO_2 metric the OECD represents about one-third of the world. It represents a smaller share of extraction and a larger share of implicit consumption, nearly twenty percent of which is imported.

In addition to the carbon accounting data, we need values for six parameters: θ, ϵ_S, ϵ_S^*, σ, σ^*, and α, the last three of which determine the demand elasticities, ϵ_D and ϵ_D^*.[32] Table 4 lists our central values for these parameters, which we have determined using a variety of sources.[33] (In our simulations, we test for sensitivity to these parameter values.) Appendix E provides additional details on our calibration procedure.

[31] The values that we take from $TECO_2$ are broadly consistent with those available from the Global Carbon Project.

[32] The eight other parameters: A, A^*, E, E^*, η, η^*, τ, and τ^* are all subsumed by calibrating to the carbon accounts.

[33] We choose $\alpha = 0.85$ based on the ratio of the value of energy used in production to value added. (In our model that ratio is $(1-\alpha)/\alpha$.) Values for ϵ_S and ϵ_S^* are estimated to

Table 3: Baseline Calibration for Home as the OECD

	Home	Foreign	Total
Home	$C_e^{HH} = 11.3$	$C_e^{HF} = 2.5$	$C_e = 13.8$
Foreign	$C_e^{FH} = 0.9$	$C_e^{FF} = 17.6$	$C_e^* = 18.5$
Total	$G_e = 12.2$	$G_e^* = 20.1$	$G_e^W = C_e^W = 32.3$
Extraction	$Q_e = 8.6$	$Q_e^* =$23.7	$Q_e^W = 32.3$

Table 4: Parameter Values

α	ϵ_S	ϵ_S^*	σ	σ^*	θ
0.85	0.5	0.5	1	1	4

6.1.3 From BAU to Optimal

For any endogenous variable x we denote the value under the optimal policy as $x(p_e, t_b, t_e)$. In the baseline the value is $x(1, 0, 0)$, denoted simply as x.[34]

Under the optimal policy, Home energy extraction is simply:

$$Q_e(p_e, t_b, t_e) = (p_e + t_b - t_e)^{\epsilon_S} Q_e,$$

for $p_e + t_b - t_e \geq \underline{a}$ and $Q_e(p_e, t_b, t_e) = 0$ otherwise. Foreign extraction is even simpler:

$$Q_e^*(p_e, t_b, t_e) = p_e^{\epsilon_S^*} Q_e^*,$$

for $p_e \geq \underline{a}^*$ and $Q_e^*(p_e, t_b, t_e) = 0$ otherwise.

The import margin remains fixed under the optimal policy while the export margin changes to:

$$\bar{j}_x(p_e, t_b, t_e) = \frac{(p_e + (1-\alpha)\, t_b)^\theta\, C_e^{FH}}{(p_e + (1-\alpha)\, t_b)^\theta\, C_e^{FH} + \left(p_e^\alpha\, (p_e + t_b)^{1-\alpha}\right)^\theta C_e^{FF}}.$$

Consumption of energy in Foreign from Foreign production is:

$$C_e^{FF}(p_e, t_b, t_e) = p_e^{-\epsilon_D^*} \left(\frac{1 - \bar{j}_x(p_e, t_b, t_e)}{1 - \bar{j}_x} \right)^{1+(1-\sigma^*)/\theta} C_e^{FF}.$$

Using similar derivations we can express the values under the optimal policy for each of the other components of energy demand and for the implicit subsidy to exporters S. See Appendix D for details.

To compute the optimal border adjustment t_b along with the equilibrium energy price p_e, we require that they clear the global energy market and satisfy (23). In particular, we require:

$$C_e^W(p_e, t_b, t_e) = Q_e^W(p_e, t_b, t_e),$$

be 0.5 using data in Asker, Collard-Wexler, and De Loecker (2018), by fitting the slope of $E(a)$ and $E^*(a)$ among oil fields with costs above the median. Appendix E provides more details. We take $\theta = 4$ based on the preferred estimate in Simonovska and Waugh (2014). We set $\underline{a} = \underline{a}^* = 10^{-6}$ so that $E(\underline{a})/E = E^*(\underline{a}^*)/E^* = 1/1000$ (given $\epsilon_S = \epsilon_S^* = 0.5$). The values for $\sigma = \sigma^* = 1$ are chosen as a compromise between a likely higher elasticity of substitution between individual goods and a lower elasticity of demand for the goods aggregate. Note that neither ϵ_D nor ϵ_D^* are particularly sensitive to this choice of σ and σ^*.

[34]While we model specific taxes, their magnitudes will have an ad-valorem interpretation relative to the baseline energy price of 1.

$$t_b = \frac{\varphi^W \epsilon_S^* Q_e^*(p_e, t_b, t_e) - \sigma^*(1-\alpha) S(p_e, t_b, t_e)}{\epsilon_S^* Q_e^*(p_e, t_b, t_e) + \epsilon_D^* C_e^{FF}(p_e, t_b, t_e)},$$

and $t_e = \varphi^W$. Our algorithm simply iterates between the first two equations, while imposing the third, until we find the vector (p_e, t_b, φ^W) that satisfies them. We follow similar procedures for the optimal constrained policies, although expressions for some of the outcomes are different, even conditional on t_b and t_e.

We can evaluate any outcome of the model at the equilibrium (p_e, t_b, φ^W) to explore the implications of the optimal policy. A key implication is the welfare benefit of the policy to Home. Our measure starts with the change in the planner's objective, $U(p_e, t_b, t_e) - U$. This term is equivalent to increased spending on services by Home, since consumption of services enters preferences linearly with price 1. To interpret the magnitude, and to make it scale free, we normalize it by Home baseline spending on goods, $C_e/(1-\alpha)$. The measure we present is thus:

$$W = \frac{1-\alpha}{C_e} \left(U(p_e, t_b, t_e) - U \right).$$

Our script is in Matlab, and we use the solving procedure described above rather than a built-in solver. Our code is available at https://github.com/dweisbach/Optimal-Unilateral-Carbon-Policy.

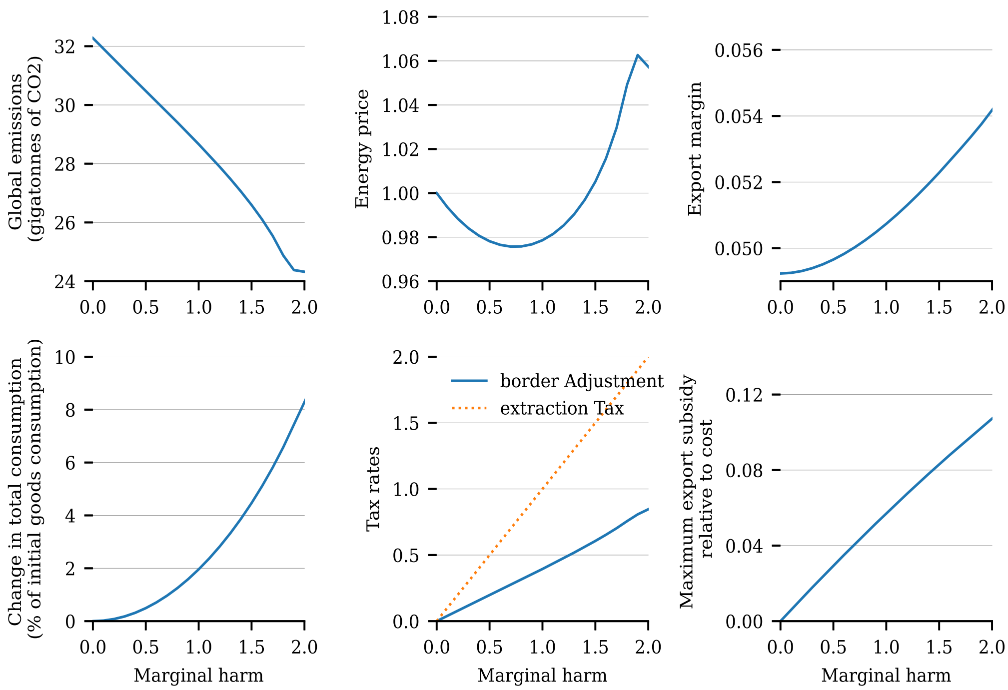
6.2 Simulations

6.2.1 Optimal Policy

We begin with a simulation of the optimal policy in the OECD (Figure 1). We illustrate the policy for marginal harm ranging from $\varphi^W = 0$ to $\varphi^W = 2$. We show (i) the emissions reductions, (ii) the change in welfare (W), (iii) the change in p_e, (iv) the tax rates under the optimal policy, (v) the change in Home's export margin, $\bar{j}_x$, and (vi) the export subsidy, S.

Global emissions go down by about ¼ when $\varphi^W = 2$, a substantial reduction given that emissions in the OECD are only about ⅓ of global emissions (as reflected in the value of G_e in Table 3). Note that the substantial reduction from the OECD policy does not mean that the OECD's emissions are near zero. Some of the reductions arise in other parts of the world because of how the optimal policy expands the carbon price to trading partners.

Figure 1: Optimal Policy in the OECD



Notably, the OECD would choose to impose a significant carbon policy even when the rest of the world does not. For $\varphi^W = 2$, the optimal carbon policy reduces global emissions by 7.6 Gt CO_2. That the OECD would choose these policies on its own may have important implications for the design of climate negotiations: even if one or more countries hold out, it makes sense for the remaining countries to impose a substantial carbon price.

The extraction tax rate (bottom middle) is always equal to φ^W. Recalling that the tax rate can be interpreted in the ad-valorem sense, the optimal tax rates range from 0 to up to twice the initial price of energy. When $\varphi = 2$ the border adjustment is just half the value of φ^W. Energy extractors are thus bearing a large share of the carbon tax. The OECD's policy, however, still pushes the energy price (top middle) below 1 until φ^W and the extraction tax approach 1.5. For even higher values of φ^W, the net price received by energy extractors in the OECD, $p_e + t_b - t_e$, approaches zero. As a result, the

OECD's extraction approaches zero as φ^W approaches 2, which can be seen in the kink in the lines for high values of φ^W (reflecting a corner solution in the simulation).

Examining the two graphs on the right-hand column of Figure 1, we can see that Home expands its export margin as marginal damages increase. By expanding its export margin, Home is able to broaden the application of its carbon policy, which becomes more important as the marginal harm from emissions increases. This feature of the policy comes at a cost that rises with φ^W.

To further examine the features of the optimal policy, we present four simulations that vary different elements of Home's policy.

6.2.2 Coalition Size

A key factor in global climate negotiations is the set of countries that will agree to emissions reductions. As noted, one of the major criticisms of the Kyoto Protocol was that it left some large emitters out of the emissions reduction coalition. The Paris Agreement was, in part, designed to address this criticism by trying to achieve universal participation.

To examine the effects of coalition size, Figure 2 shows the effects on global emissions of optimal policies with five increasingly larger coalitions, starting with just the EU and eventually going up to a globally harmonized tax.[35] Tables 5, 6 and 7 provide the calibrations for the three new scenarios. All other parameters remain the same across each case. For example, we do not adjust the energy supply elasticities based on which extractors are in the taxing coalition.

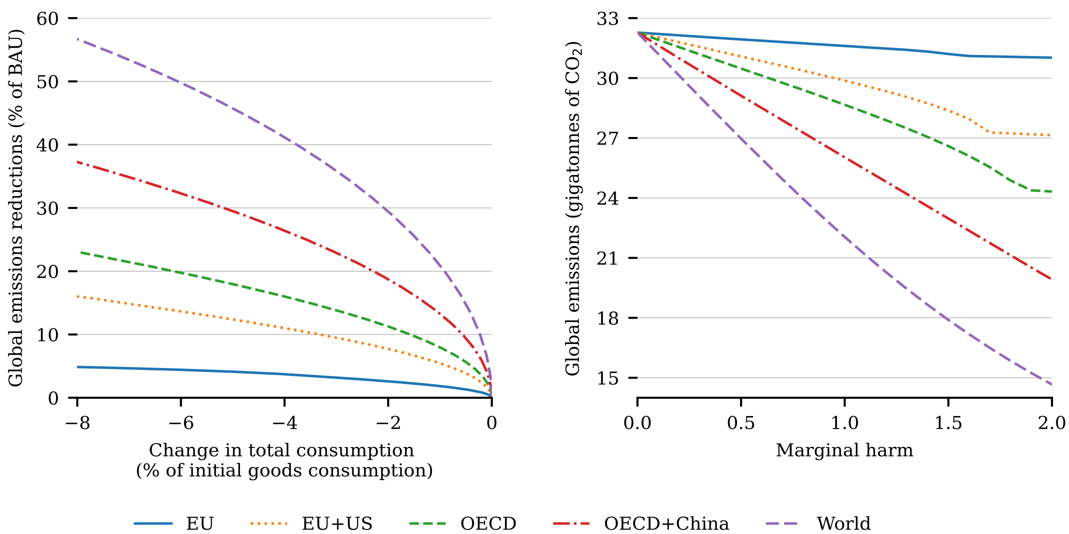
The right-hand graph shows the emissions reductions that each of the coalitions would achieve with the optimal policy for a given level of marginal harm (the analog of the upper left panel of Figure 1). The left-hand graph shows the economic cost (measured as reduced consumption) of achieving a given percentage reduction in emissions from the 2015 level (32.3 Gt CO_2).[36] Reading horizontally, it gives the effective economic cost of a given level of

[35]We treat the global case as the limit of our two-region model as Foreign becomes infinitesimally small. For the EU-only case, we treat the EU as having 28 members as it had, prior to Brexit, in 2015.

[36]Our measure of economic cost of the policy to Home starts with the welfare measure W given above, but adds $\varphi^W \left(Q_e^W(p_e, t_b, t_e) - Q_e^W \right)$ (which is negative) to the numerator. The result is necessarily a negative number, becoming more negative as a larger φ^W leads to greater emissions reductions. This measure is convenient to compute, but implicitly

emissions reductions for different taxing coalitions. Reading vertically, it gives the level of emissions reductions that would be achieved at a given economic cost for different taxing coalitions.

Figure 2: Choice of Pricing Coalition



Both figures show a consistent story, which is that there are substantial gains from expanding the taxing coalition. The EU alone has almost no power to reduce emissions. Adding the United States or the rest of the OECD countries helps significantly. Adding China to the taxing coalition further reduces the costs of any given level of reductions and increases the willingness of the coalition to reduce emissions.

Looking at the calibration tables, we can see that the size of the extraction base is the key difference between the EU and the coalition of the EU and the United States. Production and consumption roughly double, reflecting the relative size of the two economies, but extraction goes up by a factor of more than 5. With almost no extraction, the EU on its own is unable to take advantage of the extraction tax portion of the optimal policy, which means

assumes $\varphi^* = 0$. If $\varphi^* > 0$ then we overstate the economic cost to Home by ignoring transfers from Foreign to Home that offset gains to Foreign from reduced global emissions. Given a non-zero value for φ^* it is straightforward to make the necessary adjustment, which would push our measure of economic cost toward zero.

Table 5: Calibration for the European Union

	Home	Foreign	Total
Home	$C_e^{HH} = 3.0$	$C_e^{HF} = 1.0$	$C_e = 4.0$
Foreign	$C_e^{FH} = 0.5$	$C_e^{FF} = 27.8$	$C_e^* = 28.3$
Total	$G_e = 3.5$	$G_e^* = 28.8$	$G_e^W = C_e^W = 32.3$
Extraction	$Q_e = 1.0$	$Q_e^* =31.3$	$Q_e^W = 32.3$

Table 6: Calibration for the EU and the United States

	Home	Foreign	Total
Home	$C_e^{HH} = 7.7$	$C_e^{HF} = 2.0$	$C_e = 9.8$
Foreign	$C_e^{FH} = 0.7$	$C_e^{FF} = 21.8$	$C_e^* = 22.5$
Total	$G_e = 8.5$	$G_e^* = 23.8$	$G_e^W = C_e^W = 32.3$
Extraction	$Q_e = 5.4$	$Q_e^* =26.9$	$Q_e^W = 32.3$

Table 7: Calibration for the OECD plus China

	Home	Foreign	Total
Home	$C_e^{HH} = 20.1$	$C_e^{HF} = 1.7$	$C_e = 21.8$
Foreign	$C_e^{FH} = 1.4$	$C_e^{FF} = 9.1$	$C_e^{*} = 10.5$
Total	$G_e = 21.5$	$G_e^{*} = 10.8$	$G_e^{W} = C_e^{W} = 32.3$
Extraction	$Q_e = 16.24$	Q_e^{*} =16.1	$Q_e^{W} = 32.3$

that acting alone, it is ineffective at reducing emissions. Adding the United States expands the extraction base and makes the policy more effective.

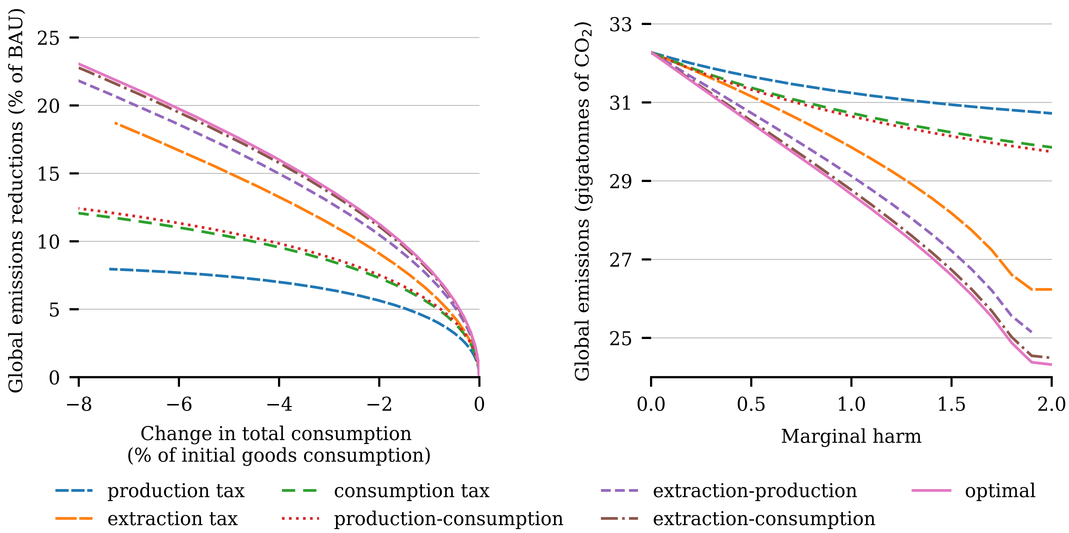
The key difference when the coalition increases to include the entire OECD and China (other than its shear increase in size) is that energy use in production and consumption in the taxing coalition are now about equal. The coalition is still a net importer of energy, however. Extraction in the coalition is half of global extraction while production and consumption are about ⅔ of the global values.

6.2.3 Choice of tax

Most analyses of carbon taxes and trade examine the effects of adding border adjustments to production taxes, shifting the tax base to domestic consumption. Returning to the assumption that the OECD is the taxing coalition, Figure 3 examines the effects of border adjustments, comparing a production tax and a production tax with optimal border adjustments to the optimal policy and to two hybrid taxes: an extraction/production hybrid and an extraction/consumption hybrid (as determined in (26)).

As can be seen, the pure production taxes do poorly whether measured by their cost per unit of emissions reduction or the level of emissions reductions that they optimally achieve. Adding border adjustments improves their performance, but only modestly. The two hybrid extraction taxes perform much better. They reduce emissions almost as much as the optimal tax (right hand figure) but do so at a somewhat greater cost (left hand figure).

Figure 3: Effects of different taxes on emissions

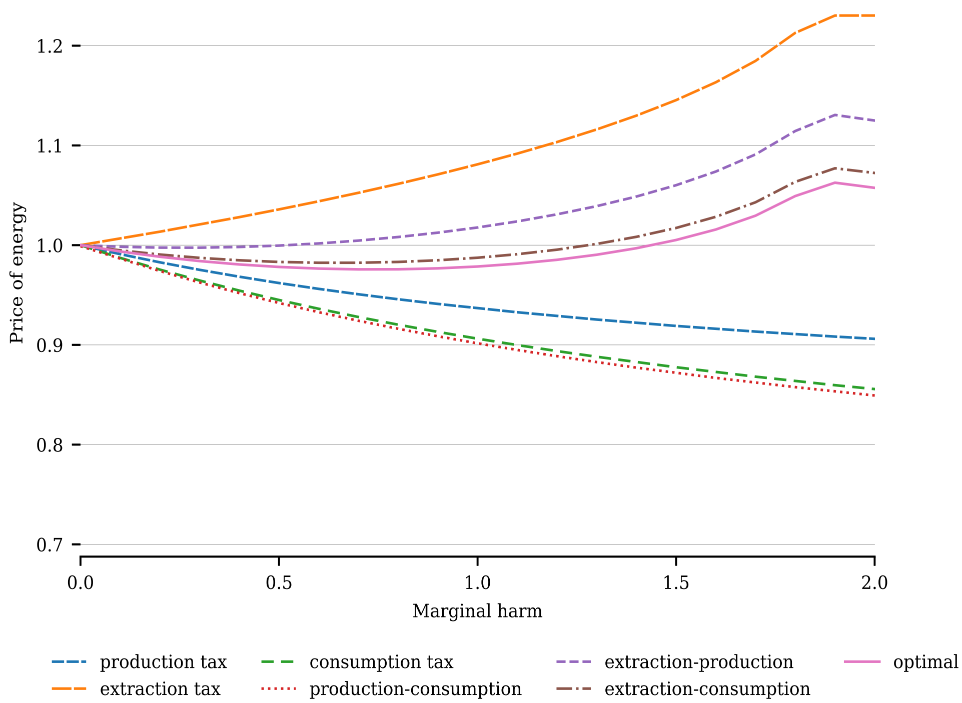


The reason that the extraction tax hybrids perform relatively well is the combination of a tax on the supply of energy and on the demand for energy allows Home to control the price of energy seen by Foreign actors. Figure 4 illustrates. It shows the change in p_e for the three "pure" taxes (extraction, production, and consumption) the optimal tax, and the two extraction tax hybrids. Both the production and consumption taxes act as demand-side taxes, reducing demand and, therefore, the price of energy transmitted to Foreign. The extraction tax acts as a tax on the supply of energy. By reduce Home's supply of energy, it increases p_e.

The extraction/production hybrids combine demand and supply-side taxes, moderating the effects on p_e. As discussed above, and illustrated in further simulations immediately below, the result is more moderate incentives in Foreign to make adjustments that offset Home's tax. The optimal tax is similar, producing a pattern very close to the extraction/consumption hybrid, although with slightly lower values of p_e. In all three cases, p_e eventually increases as the value of φ^W gets high.

As seen in Figure 3, the two extraction tax hybrids perform roughly the same. The extraction/production hybrid, however, only requires border adjustments on energy while the extraction/consumption hybrid requires border adjustments on goods as well. Border adjustments on energy would

Figure 4: Effects on p_e



be simple to implement while border adjustments on all goods would be extremely complex to implement. As a result, a clear implication of this comparison is that if the taxing coalition is constrained to pick among the conventional taxes (for example, because the optimal tax is too complex to administer or because it might run afoul of international trade law), it should choose the extraction/production hybrid. This combination produces better results than a production tax with border adjustments and roughly equivalent results to an extraction tax with border adjustments on goods, yet is easier to implement than either. The extraction/production hybrid would also be easier to implement than a conventional production tax with border adjustments on goods. As a result, the extraction/production hybrid should be strongly preferred to the conventional alternative. It both performs better and is simpler to implement.

6.2.4 Location

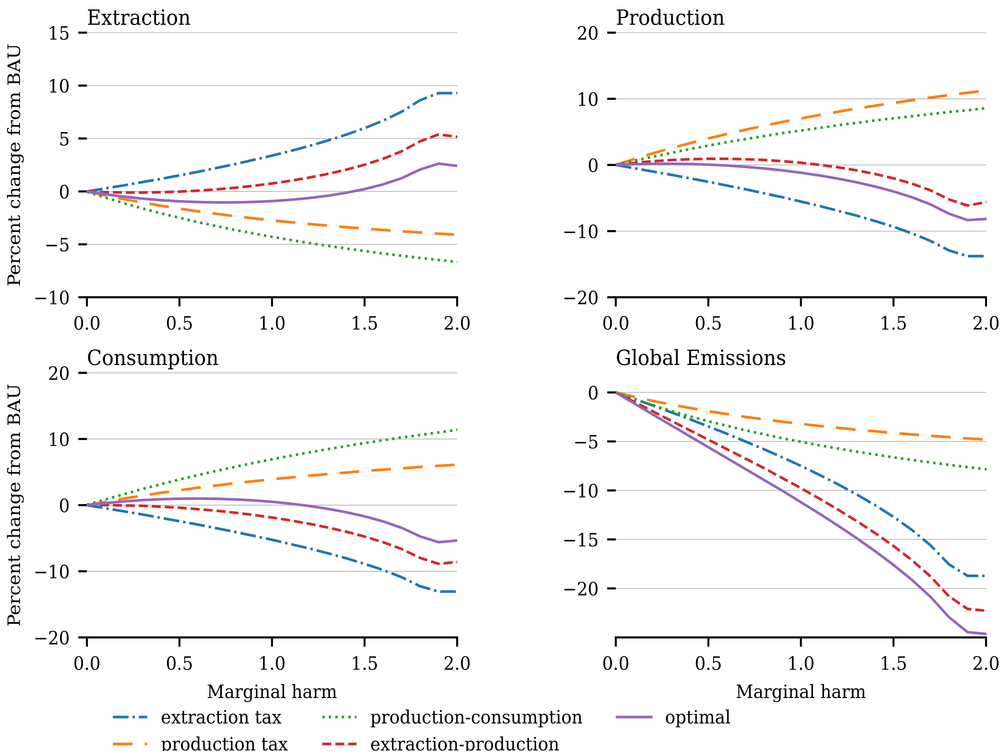
Figure 5 explores the effects of taxes on leakage and other shifts in location, focusing on how activities in Foreign change in response to Home's taxes. It illustrates five of the taxes considered in Figure 4 (dropping the extraction/consumption hybrid). It shows the percent changes in Q_e^*, G_e^*, and C_e^* relative to their values with no tax. We show, for reference in the bottom right, the change in global emissions for each of these taxes.

Changes to extraction (top left) are consistent with the changes to p_e seen in Figure 4. Extraction taxes drive up p_e and as a result, cause Foreign to increase its extraction. Production and consumption taxes drive p_e down, causing Foreign to reduce its extraction. The optimal tax and the extraction/production hybrid moderate the effects on Foreign extraction.

The opposite occurs for G_e^* and C_e^* (top right, bottom left). Because production and consumption taxes drive p_e down, G_e^* and C_e^* both go up when Home imposes those taxes. Correspondingly, Foreign production and consumption both go down when Home imposes an extraction tax. And once again, the optimal and the extraction/production hybrid operate in the middle.

A clear implication of Figures 4 and 5 is that Home's strategy in choosing its carbon tax policy should be to choose a mix of taxes that moderate the effects on p_e. Doing so allows Home to reduce shifts in location that offset domestic emissions reductions. This involves mixing an extraction tax, which raises p_e and either a production or consumption tax, which lowers it.

Figure 5: Effects on Foreign Activities



The extraction/production hybrid, in particular, has similar effects to the optimal tax but may be far simpler to implement. As a result, it is a promising avenue for further exploration.

7 Multiple Energy Sources

Up to this point we have assumed that all energy is from fossil fuel with a fixed carbon content. We could therefore normalize a unit of CO_2 to be a unit of energy, treating energy and CO_2 interchangeably. We also assumed that energy is costlessly traded, crude oil being the closest example. Here we briefly explore how our analysis can accommodate a variety of energy sources.

We introduce $K \geq 1$ sources, indexed by k, such as coal, natural gas, and solar. We assume that these sources are perfect substitutes in providing energy, but may differ in their CO_2 content, h_k. We take $k = 1$ to be crude oil, and normalize $h_1 = 1$. If k is a renewable source, $h_k = 0$. Each source of energy has a corresponding distribution of deposits, $E_k(a)$ in Home and $E_k^*(a)$ in Foreign.[37] This formulation, in terms of deposits, fits renewable sources as well since costs of generating solar, wind, and water power are also dictated by scarce geographic factors.

Our basic simplifying assumption is that the world energy market is integrated through trade in oil. Furthermore, we assume that other sources of energy are not tradable. This second assumption rules out potential policy interventions by Home to shift Foreign supply toward sources with lower CO_2 content.[38] While clearly abstracting from critical features of the energy market, these assumptions lead to a simple and intuitive generalization of our analysis above.

The quantity of energy from Home becomes:

$$Q_e = \sum_{k=1}^{K} Q_{e,k},$$

[37]In parallel to our assumptions above on $E(a)$ and $E^*(a)$, for any k we take $E_k(a)$ to be a continuous and strictly increasing function on $a \geq \underline{a}_k \geq 0$, with $E(a) = 0$ for $a < \underline{a}_k$. The same applies to $E^*(a)$, with $\underline{a}_k^* \geq 0$ replacing $\underline{a}_k$.

[38]For example, if renewables were tradable, Home might want to import them to stimulate their production in Foreign. If Home imported all that Foreign supplied, it could raise their relative price in Foreign, thus stimulating supply.

with total CO_2 content per unit of energy of:

$$h = \sum_{k=1}^{K} h_k \frac{Q_{e,k}}{Q_e}.$$

At an energy price p_e Foreign extraction is:

$$Q_e^* = \sum_{k=1}^{K} E_k^*(p_e).$$

The analysis above is the special case of $K = 1$.

7.1 Amendments to the Planner's Problem

This extension requires only a slight amendment to the planner problem. First, the planner now chooses the quantity of extraction of each type, $\{Q_{e,k}\}_{k=1}^{K}$. The inner problem is unchanged but the outer problem must be extended.

The first order condition for energy extraction from source k is:

$$\frac{\partial \mathcal{L}}{\partial Q_{e,k}} = -h_k \varphi^W - \frac{\partial L_e}{\partial Q_{e,k}} + \lambda_e \leq 0,$$

with equality if $Q_e > 0$. The extra labor in Home to extract a bit more energy from source k is the labor requirement on the marginal energy deposit for that source, $E_k^{-1}(Q_{e,k})$. If $Q_{e,k} > 0$ the first order condition therefore simplifies to:

$$Q_{e,k} = E\left(\lambda_e - h_k \varphi^W\right).$$

If $\lambda_e - h_k \varphi^W \leq \underline{a}_k$ then $Q_{e,k} = 0$, a more likely outcome if the CO_2 content h_k is high.

The first order condition for the energy price becomes:

$$\left(\frac{\eta^*}{C_g^*}\right)^{1/\sigma^*} \frac{\partial C_g^*}{\partial p_e} - \varphi^W \sum_{k=1}^{K} h_k \frac{\partial Q_e^*}{\partial p_e} - \sum_{k=1}^{K} \frac{\partial L_{e,k}^*}{\partial p_e} - \frac{\partial L_g}{\partial p_e} - \frac{\partial L_g^*}{\partial p_e}$$
$$= \lambda_e \left(\frac{\partial G_e}{\partial p_e} + \frac{\partial G_e^*}{\partial p_e} - \sum_{k=1}^{K} \frac{\partial Q_{e,k}^*}{\partial p_e}\right),$$

where:

$$\frac{\partial L_{e,k}^*}{\partial p_e} = p_e \frac{\partial E_k^*(p_e)}{\partial p_e}.$$

In its final simplified form, the condition is nearly the same as (20):

$$\lambda_e - p_e = \frac{\varphi^W \tilde{\epsilon}^*_S Q^*_e - \sigma^*(1-\alpha)S}{\tilde{\epsilon}^*_S Q^*_e + \epsilon^*_D C^{FF}_e}. \tag{31}$$

The only new ingredient is that the elasticity of Foreign CO_2 "extraction" has become:

$$\tilde{\epsilon}^*_S = \sum_{k=1}^{K} h_k \epsilon^*_{S,k} \frac{Q^*_{e,k}}{Q^*_e},$$

where:

$$\epsilon^*_{S,k} = \frac{dE^*_k}{dp_e} \frac{p_e}{E^*_k}.$$

The key insight is that a single elasticity $\tilde{\epsilon}^*_S$, which is endogenous to the policy, nonetheless captures all that is relevant about Foreign energy supply in formulating the optimal unilateral policy.

7.2 Amendments to Optimal Taxes

The optimal policy can still be implemented with an extraction tax, a border adjustment, and a subsidy to Home's marginal exporters. The last feature is unchanged by the addition of multiple energy sources.

At first blush it appears that the extraction tax must be specific to each source of energy, at a rate of $t_{e,k} = h_k \varphi^W$ per unit of energy (with $Q_{e,k}$ units in total). The policy is simpler to implement, however, with a common extraction tax of $t_e = \varphi^W$ per unit of CO_2 (with the tax applied to the $h_k Q_{e,k}$ units of CO_2 in energy from source k).

The level of the border adjustment is $t_b = \lambda_e - p_e$, given by (31). It is now applied per unit of energy, not per unit of carbon. The marginal source of energy supply in Foreign is taken into account in determining the level of the border adjustment, but not in its application. Similarly, after applying the extraction tax in Home, in which the rate depends on the CO_2 content, the price of energy to users is $p_e + t_b$ without regard to the source.

Putting the extraction tax and border adjustment together yields further insight. Extraction from source k by Home is:

$$Q_{e,k} = \max \left\{ E\left(p_e + t_b - h_k t_e\right), 0 \right\}.$$

Extraction from a low-carbon source k will be stimulated under the optimal policy if $h_k < t_b / t_e$. This inequality is satisfied for renewables.

In considering the application of carbon border adjustments, a seemingly intractable issue is whether to take account of the energy source used in producing the imported good. Kortum and Weisbach (2017) discuss this issue at length and argue that what matters is the carbon content of the marginal energy source of the country exporting the good, not the actual source of energy used to produce the good. Equation (31) formalizes that argument. The optimal unilateral policy ignores the energy source for each exporting firm, applies the border adjustment to energy content only, and sets the level of the border adjustment based on the marginal sources of Foreign's energy extraction, captured in $\tilde{\epsilon}_S^*$. What seemed like an intractable issue has a simple solution.

8 Conclusion

While the model in this paper is highly stylized, its simplicity yields analytical insights into the features of an optimal unilateral carbon policy. The main new finding is the extent to which international trade can be exploited to broaden the reach of carbon policy. The optimal carbon policy uses trade to expand carbon pricing and to lower energy use outside the narrow borders of the taxing region.

To see whether such effects are of first-order importance, it is critical to push the analysis in a more quantitative direction, extending it to multiple countries and perhaps to multiple periods of time as well. For the first extension, the multi-country model of Eaton and Kortum (2002) retains the Ricardian structure of trade in goods used here, Farrokhi and Lashkaripour (2020) consider optimal policy in a multi-country world, and the model of Larch and Wanner (2019) contains a natural multi-country extension of the energy sector. On the second extension, the dynamic analysis in Golosov, Hassler, Krusell, and Tsyvinski (2014) appears amenable to nesting within a multi-country world.

References

Asker, John, Allan Collard-Wexler, and Jan De Loecker (2019), "(Mis) Allocation, Market Power, and Global Oil Extraction," *American Economic Review*, 109, 1568-1615.

Asker, John, Allan Collard-Wexler, and Jan De Loecker (2018), "The Welfare Impact of Market Power: The OPEC Cartel," *Working Paper*, Duke University.

Balis, Kathy, Don Fullerton, and Daniel H. Karney (2014), "Negative Leakage," *Journal of the Association of Environmental and Resource Economists*, 1, 51-73.

Bernard, Alain, Carolyn Fischer, and Alan Fox (2007), "Is There a Rationale for Output-based Rebating of Environmental Levies?," *Resource and Energy Economics,* 29, 83-101.

Böhringer, Christoph, Edward J. Balistreri, and Thomas F. Rutherford (2012), "The Role of Border Carbon Adjustment in Unilateral Climate Policy: Overview of an Energy Modeling Forum study (EMF 29)," *Energy Economics*, 34. The Role of Border Carbon Adjustment in Unilateral Climate Policy: Results from EMF 29, S97–110.

Böhringer, Christoph, Andreas Lange, and Thomas F. Rutherford (2014), "Optimal Emission Pricing in the Presence of International Spillovers: Decomposing Leakage and Terms-of-Trade Motives," *Journal of Public Economics*, 110, 101-111.

Branger, Frédéric, and Philippe Quiron (2014) "Climate Policy and the 'Carbon Haven' Effect," *Wiley Interdisciplinary Reviews: Climate Change*, 5, 53-71.

Bradford, Anu (2020), The Brussels Effect, Oxford University Press, New York.

Costinot, Arnaud, Dave Donaldson, Jonathan Vogel, and Ivan Werning (2015), "Comparative Advantage and Optimal Trade Policy," *Quarterly Journal of Economics,* 659-702.

Dekle, Robert, Jonathan Eaton, and Samuel Kortum (2007), "Unbalanced Trade," *American Economic Review: Papers and Proceedings,* 97, 351-355.

Dornbusch, Rudiger, Stanley Fischer, and Paul A. Samuelson (1977), "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *American Economic Review,* 67, 823-839.

Eaton, Jonathan and Samuel Kortum (2002), "Technology, Geography, and Trade," *Econometrica*, 70, 1741-1779.

Elliott, Joshua, Ian Foster, Samuel Kortum, Todd Munson, Fernando Perez Cervantes, and David Weisbach (2010), "Trade and Carbon Taxes," *American Economic Review: Papers and Proceedings,* 100, 465-469.

Elliott, Joshua, Ian Foster, Samuel Kortum, Todd Munson, Fernando Perez Cervantes, and David Weisbach (2010), "Trade and Carbon Taxes," *American Economic Review: Papers and Proceedings,* 100, 465-469.

Elliott, Joshua, Ian Foster, Samuel Kortum, Gita K. Jush, Todd. Munson, and David Weisbach (2013), "Unilateral Carbon Taxes, Border Tax Adjustments and Carbon Leakage," *Theoretical Inquiries in Law,* 14.

EPA (2019): "Inventory of U.S. Greenhouse Gas Emissions and Sinks: 1990-2017. Annex 2 (Methodology for estimating CO2 emissions from fossil fuel combustion), Table A-42 for C coefficient and Table A-52 for heat content," *U.S. Environmental Protection Agency, Washington, DC,* U.S. EPA #430-R-19-001 (PDF).

Farrokhi, Farid and Ahmad Lashkaripour (2020), "Trade, Firm-Delocation, and Optimal Environmental Policy," unpublished, Purdue University.

Fischer, Carolyn, and Alan K. Fox (2011), "The Role of Trade and Competitiveness Measures in US Climate Policy," *American Economic Review,* 101, 258-62.

— (2012), "Comparing Policies to Combat Emissions Leakage: Border Carbon Adjustments versus Rebates," *Journal of Environmental Economics and Management*, 64, 199-216.

Fowlie, Merideth, Mar Reguant, and Stephen Ryan (2014), "Market-Based Emissions Regulation and Industry Dynamics, " *Journal of Political Economy*, 124, 249-302.

Golosov, Mikhail, John Hassler, Per Krusell, and Aleh Tsyvinski (2014), "Optimal Taxes on Fossil Fuels in General Equilibrium," *Econometrica*, 82, 41-88.

Grossman, Gene M. and Elhanan Helpman (1994), "Protection for Sale," *American Economic Review*, 84, 833-850.

Harstad, Bård (2012), Buy Coal! A Case for Supply-Side Environmental Policy," *Journal of Political Economy*, 120, 77-115.

Hemous, David. (2016), "They Dynamic Impact of Unilateral Environmental Policies," *Journal of International Economics*, 103, 80-95.

Hoel, M. (1996), "Should a Carbon Tax be Differentiated across sectors?," *Journal of Public Economics*, 59, 17–32.

Holladay, J. Scott, Mohammed Mohsin, and Shreekar Pradhan (2018), "Emissions Leakage, Environmental Policy and Trade Frictions," *Journal of Environmental Economics and Management*, 88, 95-113.

IEA (2020), "World Energy Balances," *IEA World Energy Statistics and Balances (database)*, https://doi.org/10.1787/data-00512-en.

IPCC (2008), "2006 IPCC Guidelines for National Greenhouse Gas Inventories – A primer," Prepared by the *National Greenhouse Gas Inventories Programme*, Eggleston H.S., Miwa K., Srivastava N. and Tanabe K. (eds). Published: IGES, Japan.

Jakob, Michael, Robert Marschinksi, and Michael Hubler (2013), Between a Rock and a Hard Place: A Trade-Theory Analysis of Leakage Under Production- and Consumption-Based Policies, *Environmental and Resource Economics*, 56, 47–72.

Kortum, Samuel and David Weisbach (2017), "The Design of Border Adjustments for Carbon Prices," *National Tax Journal*, 70, 421-446.

Larch, Mario and Joschka Wanner (2019), "The Consequences of Unilateral Withdrawals from the Paris Agreement," *CESifo Working Paper* 7804-2019.

Markusen, James R. (1975), "International Externalities and Optimal Tax Structures," *Journal of International Economics*, 5, 15-29.

Metcalf, Gilbert E. and David Weisbach (2009),"The Design of a Carbon Tax," *Harvard Environmental Law Review*, 33, 499-556.

Metz et al (2007), Climate Change 2007—Mitigation of Climate Change, Contribution of Working Group III to the Fourth Assessment Report of the IPCC, Cambridge University Press.

OECD (2018), Input-Output Tables. For download at http://oe.cd/i-o. Organisation for Economic Co-operation and Development, Paris.

OECD, Trade in Embodied CO2 Database (TECO2), April 2019.

Shapiro, Joseph S. and Reed Walker (2018), "Why is Pollution from US Manufacturing Declining? The Roles of Environmental Regulation, Productivity, and Trade," *American Economic Review,* 108, 3814-3854.

Simonovska, Ina and Michael Waugh (2014), "The Elasticity of Trade: Estimates and Evidence," *Journal of International Economics*, 92, 34-50.

Weitzman, Martin L. (1974), "Prices vs Quantities," *Review of Economic Studies*, 41, 477-91.

Zhang, Zhong Xiang (2012), "Competitiveness and Leakage Concerns and Border Carbon Adjustments," *International Review of Environmental and Resource Economics*, 6, 225-287.

A Global Planner's Problem

Consider a planner seeking to maximize global welfare, with control over all decisions in Foreign as well as Home. We pointed out in the paper that we can solve this problem by starting with the same Lagrangian (12) and simply enlarging the set of choice variables (while removing p_e). For convenience we repeat the Lagrangian here:

$$\begin{aligned}\mathcal{L} = &\frac{\eta^{1/\sigma}}{1-1/\sigma}\int_0^1 (y_j + m_j)^{1-1/\sigma}\,dj + \frac{(\eta^*)^{1/\sigma^*}}{1-1/\sigma^*}\int_0^1 \left(y_j^* + x_j\right)^{1-1/\sigma^*}dj - \varphi^W Q_e^W \\ &- L_e^W - \int_0^1 \left(l_j(z_j^y)y_j + \tau l_j(z_j^x)x_j + l_j^*(z_j^*)y_j + \tau^* l_j^*(z_j^m)m_j\right)dj \\ &- \lambda_e\left(\int_0^1 \left(e_j(z_j^y)y_j + \tau e_j(z_j^x)x_j + e_j^*(z_j^*)y_j^* + \tau^* e_j^*(z_j^m)m_j\right)dj - (Q_e + Q_e^*)\right).\end{aligned}$$

The global planner chooses Q_e, Q_e^*, $\{y_j\}$, $\{y_j^*\}$, $\{x_j\}$, $\{m_j\}$, $\{z_j^y\}$, $\{z_j^*\}$, $\{z_j^x\}$, and $\{z_j^m\}$ to maximize $\mathcal{L}$. At the risk of some repetition with what is in the paper, we solve this problem in detail here.

A.1 Solution

Following CDVW, we first solve the inner problem, involving conditions for an individual good given λ_e. We then turn to the outer problem, optimizing over Q_e and Q_e^* while solving for λ_e.

A.1.1 Inner Problem

Solving the inner problem consists of evaluating first order conditions with respect to the variables that are specific to some good j: y_j, y_j^*, x_j, m_j, z_j^y, z_j^*, z_j^x, and z_j^m. The Lagrangian for good j is (again, as in the paper):

$$\begin{aligned}\mathcal{L}_j = &\frac{\eta^{1/\sigma}}{1-1/\sigma}(y_j + m_j)^{1-1/\sigma} + \frac{(\eta^*)^{1/\sigma^*}}{1-1/\sigma^*}\left(y_j^* + x_j\right)^{1-1/\sigma^*} \\ &- \left(l_j(z_j^y)y_j + \tau l_j(z_j^x)x_j + l_j^*(z^*)y_j^* + \tau l_j^*(z_j^m)m_j\right) \\ &- \lambda_e\left(e_j(z_j^y)y_j + \tau e_j(z_j^x)x_j + e_j^*(z^*)y_j^* + \tau^* e_j^*(z_j^m)m_j\right).\end{aligned}$$

The first order conditions for energy intensities of production imply:

$$z_j^y = z_j^x = z_j^* = z_j^m = z = \frac{1-\alpha}{\alpha\lambda_e}.$$

The unit energy requirement in Home is thus:

$$e_j(z) = (1-\alpha) a_j \lambda_e^{-\alpha},$$

while in Foreign:

$$e_j^*(z) = (1-\alpha) a_j^* \lambda_e^{-\alpha}.$$

The FOC for y_j implies:

$$((y_j + m_j)/\eta)^{-1/\sigma} \leq a_j \lambda_e^{1-\alpha},$$

with equality if $y_j > 0$. The FOC for m_j implies:

$$((y_j + m_j)/\eta)^{-1/\sigma} \leq a_j^* \tau^* \lambda_e^{1-\alpha},$$

with equality if $m_j > 0$. The good $\bar{j}_m$ at which the FOC's for y_j and m_j both hold with equality satisfies:

$$F(\bar{j}_m) = \frac{1}{\tau^*}.$$

Thus, for $j < \bar{j}_m$:

$$y_j = \eta \left(a_j \lambda_e^{1-\alpha} \right)^{-\sigma}$$

and $m_j = 0$ while for $j > \bar{j}_m$:

$$m_j = \eta \left(a_j^* \tau^* \lambda_e^{1-\alpha} \right)^{-\sigma}$$

and $y_j = 0$.

The FOC for y_j^* implies:

$$\left(\left(y_j^* + x_j \right)/\eta^* \right)^{-1/\sigma^*} \leq a_j^* \lambda_e^{1-\alpha},$$

with equality if $y_j^* > 0$. The FOC for x_j implies:

$$\left(\left(y_j^* + x_j \right)/\eta^* \right)^{-1/\sigma^*} \leq a_j \tau \lambda_e^{1-\alpha},$$

with equality if $x_j > 0$. The good $\bar{j}_x$ at which the FOC's for y_j^* and x_j both hold satisfies:

$$F(\bar{j}_x) = \tau.$$

Since F is monotonically decreasing, it follows that $\bar{j}_x < \bar{j}_m$. For $j < \bar{j}_x$:

$$x_j = \eta^* \left(a_j \tau \lambda_e^{1-\alpha} \right)^{-\sigma^*}$$

and $y_j^* = 0$ while for $j > \bar{j}_x$:

$$y_j^* = \eta^* \left(a_j^* \lambda_e^{1-\alpha} \right)^{-\sigma^*}$$

and $x_j = 0$.

A.1.2 Implications for Aggregates

Aggregating these results from the inner problem:

$$C_e = (1-\alpha)\,\eta\left(\int_0^{\bar{j}_m} a_j^{1-\sigma} dj + (\tau^*)^{1-\sigma}\int_{\bar{j}_m}^1 \left(a_j^*\right)^{1-\sigma} dj\right)\lambda_e^{-\epsilon_D},$$

$$C_e^* = (1-\alpha)\eta^*\left(\tau^{1-\sigma^*}\int_0^{\bar{j}_x} a_j^{1-\sigma^*} dj + \int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma^*} dj\right)\lambda_e^{-\epsilon_D^*},$$

$$L_g = \alpha\eta\left(\int_0^{\bar{j}_m} a_j^{1-\sigma} dj\right)(\lambda_e)^{1-\epsilon_D} + \alpha\eta^*\left(\int_0^{\bar{j}_x} \left(\tau a_j\right)^{1-\sigma} dj\right)\lambda_e^{1-\epsilon_D^*},$$

$$L_g^* = \alpha\eta\left(\int_{\bar{j}_m}^1 (\tau^* a_j^*)^{1-\sigma} dj\right)(\lambda_e)^{1-\epsilon_D} + \alpha\eta^*\left(\int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma} dj\right)\lambda_e^{1-\epsilon_D^*},$$

$$C_g = \eta\left(\int_0^{\bar{j}_m} a_j^{1-\sigma} dj + \int_{\bar{j}_m}^1 \left(a_j^*\tau^*\right)^{1-\sigma} dj\right)^{\sigma/(\sigma-1)}\lambda_e^{-(1-\alpha)\sigma},$$

and

$$C_g^* = \eta^*\left(\int_0^{\bar{j}_x} \left(\tau a_j\right)^{1-\sigma} dj + \int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma} dj\right)^{\sigma^*/(\sigma^*-1)}\lambda_e^{-(1-\alpha)\sigma^*}.$$

These six terms are each fully determined by λ_e.

A.1.3 Outer Problem

We now turn to the optimality conditions for Q_e and Q_e^* while choosing λ_e to clear the global energy market. We can rewrite the Lagrangian in terms of aggregate magnitudes as:

$$\begin{aligned}\mathcal{L} = &\frac{\eta^{1/\sigma}}{1-1/\sigma}C_g^{1-1/\sigma} + \frac{\left(\eta^*\right)^{1/\sigma^*}}{1-1/\sigma^*}\left(C_g^*\right)^{1-1/\sigma^*} - \varphi^W\left(Q_e + Q_e^*\right)\\ &- \left(L_e + L_e^* + L_g + L_g^*\right) - \lambda_e\left(C_e + C_e^* - Q_e - Q_e^*\right).\end{aligned}$$

The first order condition with respect to Home energy extraction implies:

$$Q_e = E(\lambda_e - \varphi^W),$$

for $\lambda_e - \varphi^W \geq \underline{a}$, else $Q_e = 0$. Likewise for Foreign energy extraction:

$$Q_e^* = E^*(\lambda_e - \varphi^W),$$

for $\lambda_e - \varphi^W \geq \underline{a}^*$, else $Q_e^* = 0$. The global energy constraint determines the Lagrange multiplier as the solution:

$$C_e(\lambda_e) + C_e^*(\lambda_e) = E\left(\lambda_e - \varphi^W\right) + E^*\left(\lambda_e - \varphi^W\right).$$

A.2 Decentralized Global Optimum

We can interpret the solution in terms of a decentralized economy with a price of energy:

$$p_e = \lambda_e.$$

The global externality can be solved with an extraction tax in both countries equal to global damages:

$$t_e = t_e^* = \varphi^W.$$

Thus, energy extractors in both countries receive an after-tax price of $p_e - \varphi^W$. With a globally harmonized policy, a consumption tax at rate φ^W results in the same outcomes.[39]

A.3 Competitive Equilibrium

In a competitive equilibrium agents ignore the global externality. All outcomes other than global welfare are the same as if we simply set $\varphi^W = 0$ in the decentralized global optimum above. We treat this case as our business-as-usual baseline.

B Home Planner's Problem: Additional Details

Here we provide missing steps from Section 3 of the text, which derives the optimal unilateral policy.

[39]Inspection of the global market clearing condition for energy shows that extraction and consumption of energy remain the same if we instead set $p_e = \lambda_e + \varphi^W$. This change corresponds to adding full border adjustments, $t_b = t_b^* = \varphi^W$, to the extraction taxe, $t_e = t_e^* = \varphi^W$, which turns it into a consumption tax. Any differences in the distribution of services consumption between these two policies (a global extraction tax versus a global consumption tax) can be undone through transfers.

B.1 Energy Price

The first order condition with respect to p_e can be written as:

$$\frac{\sigma^*}{\sigma^*-1}\frac{\partial V_g^*}{\partial p_e} - \varphi^W \frac{\partial Q_e^*}{\partial p_e} - \frac{\partial L_e^*}{\partial p_e} - \frac{\partial L_g}{\partial p_e} - \frac{\partial L_g^*}{\partial p_e} = \lambda_e \left(\frac{\partial G_e}{\partial p_e} + \frac{\partial G_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right), \quad (32)$$

where we have introduced:

$$V_g^* = (\eta^*)^{1/\sigma^*} (C_g^*)^{1-1/\sigma^*}.$$

To make sense of this condition requires computing the partial derivatives (with respect to the energy price) of the seven aggregate variables that appear in (17): Q_e^*, L_e^*, G_e^*, G_e, L_g, L_g^*, and V_g^*. While we don't make it explicit in our notation that follows, all of these partial derivatives are evaluated at the optimal unilateral policy itself.

B.1.1 Dependence on the Energy Price

Foreign energy extraction depends directly on the energy price via (6), with elasticity given by (21). The response of Foreign labor employed in the energy sector is:

$$\frac{\partial L_e^*}{\partial p_e} = \frac{\partial L_e^*}{\partial Q_e^*}\frac{\partial Q_e^*}{\partial p_e} = p_e \frac{\partial Q_e^*}{\partial p_e} > 0, \quad (33)$$

with the price of energy compensating the labor required on the marginal energy deposit.

Dependence on the energy price is more subtle for the other five aggregates. Since Home directly chooses z, $\bar{j}_m$, $\bar{j}_x$, $\{m_j\}$, and $\{y_j\}$, the envelope theorem allows us treat them as fixed when differentiating the Lagrangian with respect to p_e. From the inner problem, each satisfies its own first-order condition with equality.[40] Furthermore, we can take as fixed the unit energy requirement for Home producers, whether supplying the domestic or export market. On the other hand $\{y_j^*\}$ and z^* are not chosen by the planner while for $j \in (j_0, \bar{j}_x]$ the export levels $\{x_j\}$ are optimized at a corner solution. They must be considered in the first order condition. We apply (7) and results in the bottom half of Table 2 to compute the partial derivatives of the five aggregates.

[40]Thus, C_g in (17) does not appear in (32) since it depends only on terms that were optimized in the inner problem.

Energy use by Foreign producers:

$$G_e^* = \int_{\bar{j}_x}^{1} e_j^*(z^*) y_j^* dj + \tau^* \int_{\bar{j}_m}^{1} e_j^*(z) m_j dj,$$

depends on the energy price only through the first integral, C_e^{FF} . The partial derivative we seek is therefore:

$$\frac{\partial G_e^*}{\partial p_e} = -\epsilon_D^* \frac{C_e^{FF}}{p_e} < 0. \tag{34}$$

That is, a change in the energy price affects Foreign's use of energy only through its domestic consumption C_e^{FF} and not through its exports of goods to Home C_e^{HF}. Home has chosen and optimized the determinants of C_e^{HF} ($\bar{j}_m$, m_j, and $z^m = z$).

Energy use by Home producers:

$$G_e = \int_0^{\bar{j}_m} e_j(z) y_j dj + \int_0^{j_0} \tau e_j(z) x_j dj + \int_{j_0}^{\bar{j}_x} \tau e_j(z) \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*} dj,$$

depends on the energy price only through the third term (while j_0 also depends on the energy price, its derivative adds to the second term exactly what it subtracts from the third term). The partial derivative we seek is therefore:

$$\frac{\partial G_e}{\partial p_e} = -(1-\alpha)\sigma^* \frac{1}{p_e} \int_{j_0}^{\bar{j}_x} \tau e_j(z) x_j dj. \tag{35}$$

Goods-sector employment is closely related to energy use. In Home:

$$L_g = \frac{G_e}{z} = \frac{\alpha}{1-\alpha} \lambda_e G_e,$$

so that:

$$\frac{\partial L_g}{\partial p_e} = \frac{\alpha}{1-\alpha} \lambda_e \frac{\partial G_e}{\partial p_e}. \tag{36}$$

In Foreign:

$$L_g^* = \frac{C_e^{FF}}{z^*} + \frac{C_e^{HF}}{z} = \alpha \int_{\bar{j}_x}^{1} p_j^* y_j^* dj + \frac{C_e^{HF}}{z}.$$

Since only the first term depends on the price of energy:

$$\frac{\partial L_g^*}{\partial p_e} = \alpha(1-\sigma^*) C_e^{FF}. \tag{37}$$

The new term, V_g^* can be written as:

$$V_g^* = \int_0^{j_0} a_j \tau \lambda_e^{1-\alpha} x_j dj + \int_{j_0}^{\bar{j}_x} p_j^* x_j dj + \int_{\bar{j}_x}^1 p_j^* y_j^* dj.$$

Since the first integral doesn't depend on the energy price, the derivative is:

$$\frac{\partial V_g^*}{\partial p_e} = (1-\alpha)(1-\sigma^*)\frac{1}{p_e}\int_{j_0}^{\bar{j}_x} p_j^* x_j dj + (1-\sigma^*)C_e^{FF}. \tag{38}$$

B.1.2 Restatement of the Optimality Condition

Using these partial-derivative results, we can rewrite the first order condition (32). We start by rewriting it as

$$\lambda_e \left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial G_e^*}{\partial p_e} \right) = \frac{\sigma^*}{1-\sigma^*}\frac{\partial V_g^*}{\partial p_e} + \varphi^W \frac{\partial Q_e^*}{\partial p_e} + \frac{\partial L_e^*}{\partial p_e} + \frac{\partial L_g}{\partial p_e} + \frac{\partial L_g^*}{\partial p_e} + \lambda_e \frac{\partial G_e}{\partial p_e}.$$

Next, apply (33), add $p_e \partial G_e^* / \partial p_e$ to both sides, and substitute in (36) to get:

$$(\lambda_e - p_e)\left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial G_e^*}{\partial p_e} \right) = \frac{\sigma^*}{1-\sigma^*}\frac{\partial V_g^*}{\partial p_e} + \varphi^W \frac{\partial Q_e^*}{\partial p_e} + \frac{\partial L_g^*}{\partial p_e} + p_e \frac{\partial G_e^*}{\partial p_e} + \frac{1}{1-\alpha}\lambda_e \frac{\partial G_e}{\partial p_e}.$$

Substituting in (38), (37), (35), and (34):

$$\begin{aligned}(\lambda_e - p_e)\left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial G_e^*}{\partial p_e} \right) &= \varphi^W \frac{\partial Q_e^*}{\partial p_e} + (1-\alpha)\sigma^* \frac{1}{p_e}\int_{j_0}^{\bar{j}_x} p_j^* x_j dj + \sigma^* C_e^{FF} \\ &+ \alpha(1-\sigma^*)C_e^{FF} - \epsilon_D^* C_e^{FF} - \lambda_e \sigma^* \frac{1}{p_e}\int_{j_0}^{\bar{j}_x} \tau e_j(z^x) x_j dj.\end{aligned}$$

Combining and cancelling terms:

$$(\lambda_e - p_e)\left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial G_e^*}{\partial p_e} \right) = \varphi^W \frac{\partial Q_e^*}{\partial p_e} - \frac{\sigma^*(1-\alpha)}{p_e}\int_{j_0}^{\bar{j}_x} \left(\tau a_j \lambda_e^{1-\alpha} - a_j^* p_e^{1-\alpha} \right) x_j dj.$$

We can rearrange slightly to obtain:

$$\left((\lambda_e - \varphi^W) - p_e \right)\frac{\partial Q_e^*}{\partial p_e} = (\lambda_e - p_e)\frac{\partial G_e^*}{\partial p_e} + \int_{j_0}^{\bar{j}_x} \left(\tau a_j \lambda_e^{1-\alpha} - p_j^* \right)\frac{\partial x_j}{\partial p_e} dj,$$

which is (19) in the text.

C Constrained-Optimal Policies (Unfinished)

D Solutions for Quantitative Illustration

Here we provide a list of equations for the parameterized version of the model that we use for the quantitative results in Section 5 of the paper. For each outcome, we start with the BAU competitive equilibrium value that we calibrate the model to. We then show how to express the optimal outcomes in terms of these BAU outcomes. To distinguished the two, we express outcomes under the optimal policy as functions of p_e and t_b (we leave off t_e since under the policy we always have $t = \varphi^W$). We elminate these arguments to represent BAU outcomes. Thus for an outcome x we denote the optimal outcome as $x(p_e, t_b)$ (sometimes x' for short) and the BAU outcome as simply x. Throughout we impose (27), (28), (29), and (30). .

D.1 Expressions to Compute the Optimal Policy

Most of the expressions that follow are based on integrating energy use across the continuum of goods. We start with the three expressions for unit energy requirements per good under the optimal policy:

1. For production in Home to serve consumers in Home or Foreign

$$e_j(z) = (1-\alpha)a_j(p_e + t_b)^{-\alpha}$$

2. For production in Foreign to serve consumers in Home

$$e_j^*(z) = (1-\alpha)a_j^*(p_e + t_b)^{-\alpha}$$

3. For production in Foreign to serve consumers in Foreign

$$e_j^*(z^*) = (1-\alpha)a_j^* p_e^{-\alpha}$$

These three expressions apply to BAU as well by setting $p_e = 1$ and $t_b = 0$.

What follows is a list of all the unilaterally optimal outcomes expressed in terms of p_e, t_b, and the corresponding outcomes under the BAU competitive equilibrium.

1. The import margin is invariant to the optimal policy:

$$\bar{j}'_m = \bar{j}_m(p_e, t_b) = \bar{j}_m = \frac{A}{A + (\tau^*)^{-\theta} A^*} = \frac{C^{HH}}{C_e}$$

2. Export margin:

   (a) Under unilateral optimal:

$$\bar{j}_x(p_e, t_b) = \frac{\tau^{-\theta} A p_e^{-\alpha\theta} (p_e + t_b)^{-(1-\alpha)\theta}}{\tau^{-\theta} A p_e^{-\alpha\theta} (p_e + t_b)^{-(1-\alpha)\theta} + A^* (p_e + (1-\alpha) t_b)^{-\theta}}$$

   (b) Under BAU:

$$\bar{j}_x = \frac{\tau^{-\theta} A}{\tau^{-\theta} A + A^*} = \frac{C_e^{FH}}{C_e^*}$$

   (c) Expressed in terms of BAU:

$$\bar{j}_x(p_e, t_b) = \frac{\bar{j}_x p_e^{-\alpha\theta} (p_e + t_b)^{-(1-\alpha)\theta}}{\bar{j}_x p_e^{-\alpha\theta} (p_e + t_b)^{-(1-\alpha)\theta} + (1 - \bar{j}_x) (p_e + (1-\alpha) t_b)^{-\theta}}$$

   (d) Shorthand:

$$\bar{j}_x{}' = \bar{j}_x(p_e, t_b).$$

3. Intermediate export margin:

   (a) Under unilateral optimal:

$$j_0(p_e, t_b) = \frac{\tau^{-\theta} A (p_e + t_b)^{-(1-\alpha)\theta}}{\tau^{-\theta} A (p_e + t_b)^{-(1-\alpha)\theta} + A^* p_e^{-(1-\alpha)\theta}}$$

   (b) Under BAU:

$$j_0 = \bar{j}_x = \frac{\tau^{-\theta} A}{\tau^{-\theta} A + A^*} = \frac{C_e^{FH}}{C_e^*}$$

   (c) Expressed in terms of BAU:

$$j_0(p_e, t_b) = \frac{j_0 (p_e + t_b)^{-(1-\alpha)\theta}}{j_0 (p_e + t_b)^{-(1-\alpha)\theta} + (1 - j_0) p_e^{-(1-\alpha)\theta}}$$

(d) Shorthand:

$$j_0' = j_0(p_e, t_b).$$

4. Energy used by producers in Home to supply Home consumers:

   (a) Under unilateral optimal:

$$\begin{aligned} C_e^{HH}(p_e, t_b) &= \int_0^{\bar{j}_m(p_e,t_b)} e_j(z) y_j dj \\ &= \eta \left(1-\alpha\right) \left(p_e + t_b\right)^{-\epsilon_D} \int_0^{\bar{j}_m(p_e,t_b)} a_j^{1-\sigma} dj \\ &= \eta \left(1-\alpha\right) \left(p_e + t_b\right)^{-\epsilon_D} \frac{A^{(\sigma-1)/\theta}}{1+\left(1-\sigma\right)/\theta} \left(\bar{j}_m'\right)^{1+(1-\sigma)/\theta} \end{aligned}$$

   (b) Under BAU:

$$C_e^{HH} = \eta \left(1-\alpha\right) \frac{A^{(\sigma-1)/\theta}}{1+\left(1-\sigma\right)/\theta} \left(\bar{j}_m\right)^{1+(1-\sigma)/\theta}$$

   (c) Expressed in terms of BAU:

$$C_e^{HH}(p_e, t_b) = \left(p_e + t_b\right)^{-\epsilon_D} C_e^{HH}$$

5. Energy used by producers in Home to supply exports of Home:

(a) Under unilateral optimal:

$$
\begin{aligned}
C_e^{FH}(p_e,t_b) &= C_e^{FH,1}(p_e,t_b) + C_e^{FH,2}(p_e,t_b)\\
C_e^{FH,1}(p_e,t_b) &= \tau \int_0^{j_0(p_e,t_b)} e_j(z)x_j dj\\
&= \tau^{1-\sigma^*}\eta^*(1-\alpha)(p_e+t_b)^{-\epsilon_D^*}\int_0^{j_0(p_e,t_b)} \left(a_j^*\right)^{1-\sigma^*} dj\\
&= \tau^{1-\sigma^*}\eta^*(1-\alpha)(p_e+t_b)^{-\epsilon_D^*}\frac{A^{-(1-\sigma^*)/\theta}}{1+(1-\sigma^*)/\theta}\left(j_0'\right)^{1+(1-\sigma^*)/\theta}\\
C_e^{FH,2}(p_e,t_b) &= \tau \int_{j_0(p_e,t_b)}^{\bar{j}_x(p_e,t_b)} e_j(z)x_j dj\\
&= \tau\eta^*(1-\alpha)p_e^{(\alpha-1)\sigma^*}(p_e+t_b)^{-\alpha}\int_{j_0(p_e,t_b)}^{\bar{j}_x(p_e,t_b)} a_j\left(a_j^*\right)^{-\sigma^*} dj\\
&= \tau\eta^*(1-\alpha)p_e^{(\alpha-1)\sigma^*}(p_e+t_b)^{-\alpha}\frac{(A^*)^{\sigma^*/\theta}}{A^{1/\theta}}\\
&\left(B\left(\bar{j}_x', \frac{1+\theta}{\theta}, \frac{\theta-\sigma^*}{\theta}\right) - B\left(j_0', \frac{1+\theta}{\theta}, \frac{\theta-\sigma^*}{\theta}\right)\right)
\end{aligned}
$$

where and $B(x,a,b)$ is the incomplete beta function[41]

(b) Under BAU:

$$
C_e^{FH} = \tau^{1-\sigma^*}\eta^*(1-\alpha)\frac{A^{(\sigma^*-1)/\theta}}{1+(1-\sigma^*)/\theta}\left(\bar{j}_x\right)^{1+(1-\sigma^*)/\theta}
$$

[41]The incomplete beta function is:

$$
B(x,a,b) = \int_0^x i^{a-1}(1-i)^{b-1} di,
$$

for $0 \le x \le 1$, $a > 0$, and $b > 0$. Setting $x = 1$ gives the beta function itself, $B(a,b)$.

(c) Expressed in terms of BAU:

$$C_e^{FH,1}(p_e, t_b) = (p_e + t_b)^{-\epsilon_D^*} \left(\frac{j_0(p_e, t_b)}{j_0} \right)^{1+(1-\sigma^\star)/\theta} C_e^{FH}$$

$$C_e^{FH,2}(p_e, t_b) = \tau^{\sigma^*} \frac{\theta + 1 - \sigma^*}{\theta} p_e^{-\epsilon_D^*} \left(\frac{p_e + t_b}{p_e} \right)^{-\alpha} \left(\frac{A^*}{A} \right)^{\sigma^*/\theta}$$
$$\frac{\left(B\left(\bar{j}_x', \frac{1+\theta}{\theta}, \frac{\theta-\sigma^*}{\theta}\right) - B\left(j_0', \frac{1+\theta}{\theta}, \frac{\theta-\sigma^*}{\theta}\right)\right)}{\bar{j}_x^{1+(1-\sigma^*)/\theta}} C_e^{FH}$$
$$= \frac{\theta + 1 - \sigma^*}{\theta} \left(\frac{1 - \bar{j}_x}{\bar{j}_x} \right)^{\sigma^*/\theta} p_e^{-\epsilon_D^*} \left(\frac{p_e + t_b}{p_e} \right)^{-\alpha}$$
$$\frac{\left(B\left(\bar{j}_x', \frac{1+\theta}{\theta}, \frac{\theta-\sigma^*}{\theta}\right) - B\left(j_0', \frac{1+\theta}{\theta}, \frac{\theta-\sigma^*}{\theta}\right)\right)}{\bar{j}_x^{1+(1-\sigma^*)/\theta}} C_e^{FH}$$

6. Energy used by producers in Foreign to supply Foreign consumers:

(a) Under unilateral optimal:

$$C_e^{FF}(p_e, t_b) = \int_{\bar{j}_x(p_e,t_b)}^{1} e_j^*(z^*) y_j^* dj$$
$$= \eta^*(1-\alpha) p_e^{-\epsilon_D^*} \int_{\bar{j}_x(p_e,t_b)}^{1} \left(a_j^*\right)^{1-\sigma^*} dj$$
$$= \eta^*(1-\alpha) p_e^{-\epsilon_D^*} \frac{(A^*)^{(\sigma^*-1)/\theta}}{1 + (1-\sigma^*)/\theta} \left(1 - \bar{j}_x'\right)^{1+(1-\sigma^*)/\theta}$$

(b) Under BAU:

$$C_e^{FF} = \eta^*(1-\alpha) \frac{(A^*)^{(\sigma^*-1)/\theta}}{1 + (1-\sigma^*)/\theta} \left(1 - \bar{j}_x\right)^{1+(1-\sigma^*)/\theta}$$

(c) Expressed in terms of BAU:

$$C_e^{FF}(p_e, t_b) = p_e^{-\epsilon_D^*} \left(\frac{1 - \bar{j}_x'}{1 - \bar{j}_x} \right)^{1+(1-\sigma^*)/\theta} C_e^{FF}$$

7. Energy used by producers in Foreign to supply imports of Home:

(a) Under unilateral optimal:

$$
\begin{aligned}
C_e^{HF}(p_e,t_b) &= \tau^* \int_{\bar{j}_m(p_e,t_b)}^{1} e_j^*(z) m_j dj \\
&= (\tau^*)^{1-\sigma} \eta(1-\alpha)(p_e+t_b)^{-\epsilon_D} \int_{\bar{j}_m(p_e,t_b)}^{1} \left(a_j^*\right)^{1-\sigma} dj \\
&= (\tau^*)^{1-\sigma} \eta(1-\alpha)(p_e+t_b)^{-\epsilon_D} \frac{(A^*)^{(\sigma-1)/\theta}}{1+(1-\sigma)/\theta} (1-\bar{j}_m')^{1+(1-\sigma)/\theta}
\end{aligned}
$$

(b) Under BAU:

$$
C_e^{HF} = (\tau^*)^{1-\sigma} \eta(1-\alpha) \frac{(A^*)^{(\sigma-1)/\theta}}{1+(1-\sigma)/\theta} (1-\bar{j}_m)^{1+(1-\sigma)/\theta}
$$

(c) Expressed in terms of BAU:

$$
C_e^{HF}(p_e,t_b) = (p_e+t_b)^{-\epsilon_D} C_e^{HF}
$$

8. Value of Home exports of goods:

(a) Under unilateral optimal:

$$
\begin{aligned}
V_g^{FH} &= V_g^{FH,1}(p_e,t_b) + V_g^{FH,2}(p_e,t_b) \\
V_g^{FH,1}(p_e,t_b) &= \int_0^{j_0(p_e,t_b)} p_j^x x_j dj \\
&= \eta^* (p_e+t_b)^{1-\epsilon_D^*} \int_0^{j_0(p_e,t_b)} a_j^{1-\sigma^*} dj \\
&= \tau^{1-\sigma^\star} \eta^\star (p_e+t_b)^{1-\epsilon_D^\star} \frac{A^{-(1-\sigma^\star)/\theta}}{1+(1-\sigma^\star)/\theta} (j_0')^{1+(1-\sigma^\star)/\theta} \\
V_g^{FH,2}(p_e,t_b) &= \int_{j_0(p_e,t_b)}^{\bar{j}_x(p_e,t_b)} p_j^x x_j dj \\
&= \eta^* p_e^{1-\epsilon_D^*} \int_{j_0(p_e,t_b)}^{\bar{j}_x(p_e,t_b)} \left(a_j^*\right)^{1-\sigma^*} dj \\
&= \eta^* p_e^{1-\epsilon_D^*} \frac{(A^*)^{(\sigma^*-1)/\theta}}{1+(1-\sigma^*)/\theta} \\
&\quad \left((1-j_0')^{(\theta+1-\sigma^*)/\theta} - (1-\bar{j}_x')^{(\theta+1-\sigma^*)/\theta}\right)
\end{aligned}
$$

(b) Under BAU:

$$V_g^{FH} = \tau^{1-\sigma^*}\eta^* \frac{A^{(\sigma^*-1)/\theta}}{1+(1-\sigma^*)/\theta}\left(\bar{j}_x\right)^{1+(1-\sigma^*)/\theta}$$

(c) Expressed in terms of BAU:

$$V_g^{FH,1}(p_e,t_b) = (p_e+t_b)^{1-\epsilon_D^*}\left(\frac{j_0'}{j_0}\right)^{1+(1-\sigma^\star)/\theta} V_g^{FH}$$

$$V_g^{FH,2}(p_e,t_b) = p_e^{1-\epsilon_D^*}\frac{\left((1-j_0')^{(\theta+1-\sigma^*)/\theta}-(1-\bar{j}_x')^{(\theta+1-\sigma^*)/\theta}\right)}{\bar{j}_x\left(1-\bar{j}_x\right)^{(1-\sigma^\star)/\theta}} V_g^{FH}$$

Substitute in $V_g^{FH} = \frac{1}{1-\alpha}C_e^{FH}$:

$$V_g^{FH,1}(p_e,t_b) = (p_e+t_b)^{1-\epsilon_D^*}\left(\frac{j_0'}{j_0}\right)^{1+(1-\sigma^\star)/\theta}\frac{1}{1-\alpha}C_e^{FH}$$

$$V_g^{FH,2}(p_e,t_b) = p_e^{1-\epsilon_D^*}\frac{\left((1-j_0')^{(\theta+1-\sigma^*)/\theta}-(1-\bar{j}_x')^{(\theta+1-\sigma^*)/\theta}\right)}{\bar{j}_x\left(1-\bar{j}_x\right)^{(1-\sigma^\star)/\theta}}\frac{1}{1-\alpha}C_e^{FH}$$

9. Value of Home's imports of goods:

(a) Under unilateral optimal:

$$\begin{aligned} V_g^{HF}(p_e,t_b) &= \int_{\bar{j}_m(p_e,t_b)}^1 p_j^m m_j dj = \int_{\bar{j}_m}^1 p_j^m m_j dj \\ &= (\tau^*)^{1-\sigma}\eta\,(p_e+t_b)^{1-\epsilon_D}\left(\frac{p_e+\alpha t_b}{p_e+t_b}\right)\int_{\bar{j}_m}^1 \left(a_j^*\right)^{1-\sigma} dj \\ &= (\tau^*)^{1-\sigma}\eta\,(p_e+t_b)^{1-\epsilon_D}\left(\frac{p_e+\alpha t_b}{p_e+t_b}\right)\frac{(A^*)^{(\sigma-1)/\theta}}{1+(1-\sigma)/\theta}\left(1-\bar{j}_m\right)^{1+(1-\sigma)/\theta} \end{aligned}$$

(b) Under BAU:

$$V_g^{HF} = (\tau^*)^{1-\sigma}\eta\frac{(A^*)^{(\sigma-1)/\theta}}{1+(1-\sigma)/\theta}\left(1-\bar{j}_m\right)^{1+(1-\sigma)/\theta}$$

(c) Expressed in terms of BAU:

$$V_g^{HF}(p_e, t_b) = (p_e + t_b)^{-\epsilon_D} (p_e + \alpha t_b) V_g^{HF}$$

Substitute in $V_g^{HF} = \frac{1}{1-\alpha} C_e^{HF}$:

$$V_g^{HF}(p_e, t_b) = (p_e + t_b)^{-\epsilon_D} (p_e + \alpha t_b) \frac{1}{1-\alpha} C_e^{HF}$$

10. Energy extraction by Home:

(a) Under unilateral optimal:

$$Q_e(p_e, t_b) = \left(p_e + t_b - \varphi^W\right)^{\epsilon_S} E$$

(b) Under BAU:

$$Q_e = E$$

(c) Expressed in terms of BAU:

$$Q_e(p_e, t_b) = \left(p_e + t_b - \varphi^W\right)^{\epsilon_S} Q_e$$

11. Energy extraction by Foreign:

(a) Under unilateral optimal:

$$Q_e^*(p_e) = (p_e)^{\epsilon_S^*} E^*$$

(b) Under BAU:

$$Q_e^* = E^*$$

(c) Expressed in terms of BAU:

$$Q_e^*(p_e) = (p_e)^{\epsilon_S^*} Q_e^*$$

D.2 Expressions to Compute Welfare

Having solved for the optimal border adjustment and the corresponding change in the global energy price we can compute all other outcomes as well. A key outcome is Home's welfare in moving to the optimal unilateral policy from the BAU competitive equilibrium.

Home's Utility (dropping constants) can be expressed as:

1. Under BAU:

$$\begin{aligned}U &= \frac{\sigma}{\sigma-1}\eta^{1/\sigma}C_g^{1-1/\sigma} + \frac{\sigma^*}{\sigma^*-1}(\eta^*)^{1/\sigma^*}(C_g^*)^{1-1/\sigma} \\ &\quad - \varphi^W(Q_e+Q_e^*) - L_g - L_g^* - L_e - L_e^* \\ &= \frac{\sigma}{\sigma-1}V_g + \frac{\sigma^*}{\sigma^*-1}V_g^* - \varphi^W Q_e^W - L_g^W - L_e^W\end{aligned}$$

2. Under unilateral optimal:

$$U(p_e,t_b) = \frac{\sigma}{\sigma-1}V_g(p_e,t_b) + \frac{\sigma^*}{\sigma^*-1}V_g^*(p_e,t_b) - \varphi^W Q_e^W(p_e,t_b) - L_g^W(p_e,t_b) - L_e^W(p_e,t_b)$$

3. The change in moving to the optimal unilateral policy from the BAU competitive equilibrium:

$$\begin{aligned}U(p_e,t_b) - U &= \frac{\sigma}{\sigma-1}(V_g(p_e,t_b) - V_g) + \frac{\sigma^*}{\sigma^*-1}(V_g^*(p_e,t_b) - V_g^*) \\ &\quad - \varphi^W(Q_e^W(p_e,t_b) - Q_e^W) - (L_g^W(p_e,t_b) - L_g^W) - (L_e^W(p_e,t_b) - L_e^W)\end{aligned}$$

   Our preferred measure of welfare is normalized by BAU spending on goods:

$$W = \frac{U(p_e,t_b) - U}{V_g}$$

For the terms in the welfare function above, we show:

1. Home's employment in energy extraction:

   (a) Change from BAU to unilateral optimal:

$$\begin{aligned}L_e(p_e,t_b) - L_e &= \int_1^{p_e+t_b-\varphi^W} a\, dE(a) \\ &= Q_e \int_1^{p_e+t_b-\varphi^W} \epsilon_S a^{\epsilon_S} da \\ &= \frac{\epsilon_S}{\epsilon_S+1}((p_e+t_b-\varphi^W)^{\epsilon_S+1} - 1)Q_e\end{aligned}$$

2. Foreign's employment in energy extraction:

(a) Change from BAU to unilateral optimal:

$$\begin{aligned} L_e^*(p_e, t_b) - L_e^* &= \int_1^{p_e} a^* \, dE^*(a^*) \\ &= Q_e^* \int_1^{p_e} \epsilon_S^* \, (a^*)^{\epsilon_S^*} \, da^* \\ &= \frac{\epsilon_S^*}{\epsilon_S^* + 1} (p_e^{\epsilon_S^* + 1} - 1) Q_e^* \end{aligned}$$

3. Labor employed in production in Home:

(a) Under unilateral optimal:

$$L_g(p_e, t_b) = \frac{\alpha}{1-\alpha} (p_e + t_b) \left(C_e^{HH}(p_e, t_b) + C_e^{FH}(p_e, t_b) \right)$$

(b) Under BAU:

$$L_g = \frac{\alpha}{1-\alpha} \left(C_e^{HH} + C_e^{FH} \right)$$

(c) Change from BAU to unilateral optimal:

$$L_g(p_e, t_b) - L_g = \frac{\alpha}{1-\alpha} \left((p_e + t_b) \left(C_e^{HH}(p_e, t_b) + C_e^{FH}(p_e, t_b) \right) - C_e^{HH} - C_e^{FH} \right)$$

4. Labor employed in production in Foreign:

(a) Under unilateral optimal:

$$L_g^*(p_e, t_b) = \frac{\alpha}{1-\alpha} \left((p_e + t_b) \, C_e^{HF}(p_e, t_b) + p_e C_e^{FF}(p_e, t_b) \right)$$

(b) Under BAU:

$$L_g^* = \frac{\alpha}{1-\alpha} \left(C_e^{HF} + C_e^{FF} \right)$$

(c) Change from BAU to unilateral optimal:

$$L_g^*(p_e, t_b) - L_g^* = \frac{\alpha}{1-\alpha} \left((p_e + t_b) \, C_e^{HF}(p_e, t_b) - C_e^{HF} + p_e C_e^{FF}(p_e, t_b) - C_e^{HF} \right)$$

5. The value of Home's spending on goods:

(a) Under unilateral optimal:

$$V_g(p_e,t_b) = \eta^{1/\sigma} C_g(p_e,t_b)^{1-1/\sigma} = \frac{1}{1-\alpha}(p_e+t_b)\left(C_e^{HH}(p_e,t_b)+C_e^{HF}(p_e,t_b)\right)$$
$$= \eta\,(p_e+t_b)^{1-\epsilon_D}\,\frac{\left(A+(\tau^*)^{-\theta}A^*\right)^{(\sigma-1)/\theta}}{1+(1-\sigma)/\theta}$$

(b) Under BAU:

$$V_g = \eta\frac{\left(A+(\tau^*)^{-\theta}A^*\right)^{(\sigma-1)/\theta}}{1+(1-\sigma)/\theta}$$

(c) Expressed in terms of BAU:

$$V_g(p_e,t_b) = (p_e+t_b)^{1-\epsilon_D}\,V_g$$

Substitute in $V_g = \frac{1}{1-\alpha}C_e$:

$$V_g(p_e,t_b) = (p_e+t_b)^{1-\epsilon_D}\,\frac{1}{1-\alpha}C_e$$

6. The term that enters the change in Home's welfare is:

$$\frac{\sigma}{\sigma-1}(V_g(p_e,t_b)-V_g) = V_g\frac{\left((p_e+t_b)^{(1-\alpha)(1-\sigma)}-1\right)}{(\sigma-1)/\sigma}$$

For the case of $\sigma = 1$ this term reduces to:

$$\lim_{\sigma\to 1} V_g\frac{(p_e+t_b)^{(1-\alpha)(1-\sigma)}}{(\sigma-1)/\sigma} = -(1-\alpha)V_g\ln(p_e+t_b) = -C_e\ln(p_e+t_b)$$

7. The value of Foreign's spending on goods:

(a) Under unilateral optimal:

$$V_g^*(p_e,t_b) = (\eta^*)^{1/\sigma^*}(C_g^*(p_e,t_b))^{1-1/\sigma^*}$$
$$= V_g^{FH}(p_e,t_b)+V_g^{FF}(p_e,t_b)$$
$$= \eta^*\frac{\left(\tau^{-\theta}A(p_e+t_b)^{-(1-\alpha)\theta}+A^*p_e^{-(1-\alpha)\theta}\right)^{-(1-\sigma^*)/\theta}}{1+(1-\sigma^*)/\theta}$$

(b) Under BAU:

$$V_g^* = \eta^* \frac{\left(A^* + \tau^{-\theta} A\right)^{(\sigma^*-1)/\theta}}{1 + (1-\sigma^*)/\theta}$$

(c) Expressed in terms of BAU:

$$\begin{aligned} V_g^*(p_e, t_b) &= V_g^* \left(\frac{j_0}{j_0(p_e, t_b)} (p_e + t_b)^{-(1-\alpha)\theta} \right)^{-(1-\sigma^*)/\theta} \\ &= V_g^* \left(\frac{j_0}{j_0(p_e, t_b)} \right)^{-(1-\sigma^*)/\theta} (p_e + t_b)^{1-\epsilon_D^*} \end{aligned}$$

8. The term that enters the change in Foreign's welfare is:

$$\frac{\sigma^*}{\sigma^* - 1}(V_g^*(p_e, t_b) - V_g^*) = V_g^* \frac{\left(\left(\frac{j_0}{j_0(p_e,t_b)} \right)^{-(1-\sigma^*)/\theta} (p_e + t_b)^{1-\epsilon_D^*} - 1 \right)}{(\sigma^* - 1)/\sigma^*}$$

For the case of $\sigma^* = 1$ this term reduces to:

$$\lim_{\sigma^* \to 1} V_g^* \frac{\left(\left(\frac{j_0}{j_0(p_e,t_b)} \right)^{-(1-\sigma^*)/\theta} (p_e + t_b)^{1-\epsilon_D^*} - 1 \right)}{(\sigma^* - 1)/\sigma^*} = V_g^* \frac{1}{\theta} ln \left(\frac{j_0}{j_0(p_e, t_b)} (p_e + t_b)^{-(1-\alpha)\theta} \right)$$

9. Global emissions:

(a) Under BAU:

$$Q_e^W = Q_e + Q_e^*$$

(b) Under unilateral optimal:

$$Q_e^W(p_e, t_b) = Q_e(p_e, t_b) + Q_e^*(p_e)$$

E Data and Calibration

E.1 Calibration

For our quantitative analysis we calibrate the model to fossil fuel extraction and the energy embodied in trade between the region that, in our model, will enact a carbon policy (Home) and the region that will remain with business as usual (Foreign). Our common unit for energy is gigatonnes of CO_2, based on the quantity released by its combustion.

We consider three scenarios for the regions representing Home and Foreign. In the first, the United States is Home and all other countries are Foreign. The alternative scenarios, respectively, are the European Union prior to Brexit (EU28) as Home (and all other countries as Foreign) and the members of the Organization for Economic Cooperation and Development (OECD37) as Home (and all others as Foreign).

Our data source for energy consumption is The Trade in Embodied CO_2 (TECO2) database from OECD. We use their measure of consumption-based CO_2 emissions embodied in domestic final demand and the country of origin of emissions. This database covers 83 countries and regional groups over the period 2005-2015. Carbon dioxide embodied in world consumption in 2015 is 32.78 gigatonnes. We cross-checked the results with a dataset from the Global Carbon Project. The overall difference is less than ten percent.

Extraction data are from the International Energy Agency (IEA), which provides the World Energy Statistics Database on energy supply from all energy sources, including fossil fuels, biofuels, hydro, geothermal, renewables and waste. This dataset covers 143 countries as well as regional and world totals. The data are provided in units of kilotonnes of oil equivalent (ktoe). In order to keep the units consistent with the energy consumption data (gigatonnes of carbon dioxide), we first convert to terajoules (TJ) (1 ktoe = 41.868 TJ) and then apply emission factors to the five fossil fuel types to calculate CO_2 emissions. The five fossil fuel types considered are coal and coal products, natural gas, peat and peat products, oil products, as well as crude, NGL and feedstocks. The emission factors are default emission factors for stationary combustion from the 2006 IPCC Guidelines for National Greenhouse Gas Inventories. To be specific, we convert 1 TJ of crude, NGL and feedstocks to 73,300 kg CO_2, 1 TJ of natural gas to $56,100$ kg CO_2, and 1 TJ of coal, peat and oil products to $94,600$ kg CO_2. Using this calculation, world extraction is 35.96 gigatonnes of carbon dioxide.

To explain the discrepancy between world consumption and world extraction, note that the OECD data for embodied carbon does not include non-energy use of fossil fuels. In other words, some fossil fuels extracted are not combusted to produce energy. Instead, they are consumed directly or as intermediate goods. For example, petroleum can be used as asphalt and road oil and as petrochemical feedstocks for agricultural land. However, given that combusted energy is the source of CO_2 emissions, non-energy use of fossil fuel extraction is excluded in our analysis.

To make this adjustment, we note that, according to EIA (2018), approximately 8 percent of fossil fuels are not combusted in the United States. Applying this rate to the world extraction, we get a number close to world consumption ($35.96 * 0.92 = 33.08$, vs. 32.78). Thus, we can simply re-scale the world extraction data so that world extraction is equal to world consumption. To be specific, the original extraction data is divided by 1.097 (the ratio of world extraction to world consumption). Tables 3, 5, 6, and 7 display the resulting data we use for our calibration.

E.2 Parameter Values

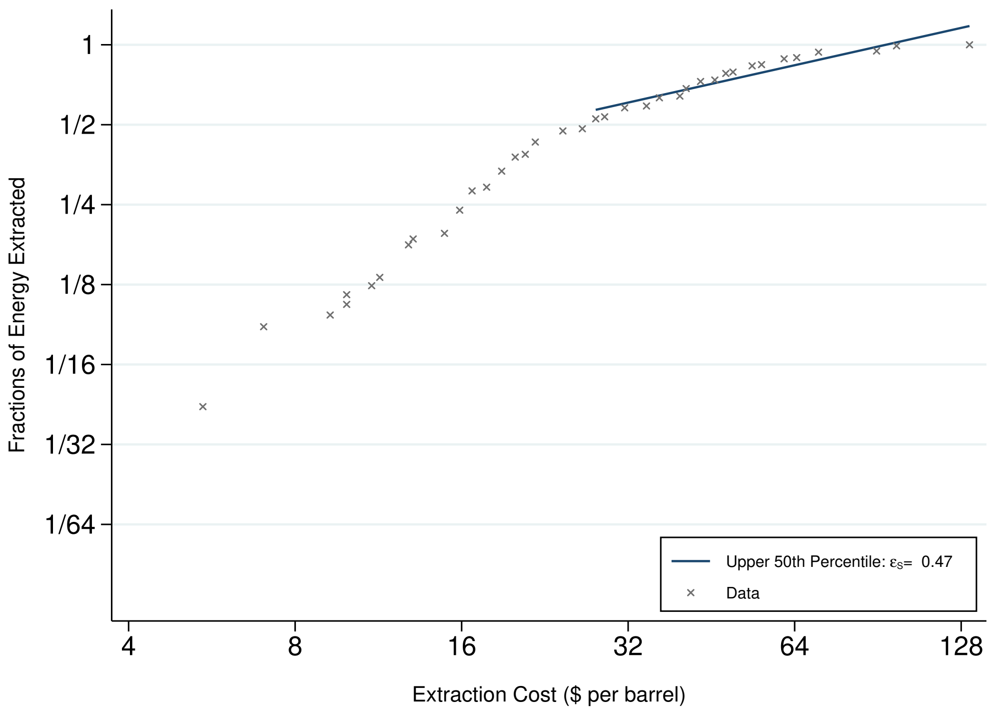
For the key parameter in the goods production function α, the output elasticity of labor, we calibrate $(1-\alpha)/\alpha$ to the value of energy used in production $p_e G_e$ relative to the value added.[42] The data from TECO2 records the carbon emissions embodied by sector and country. We can convert to barrels of oil based on 0.43 metric tons of CO_2 per barrel of crude oil (from EPA, 2019). The price per barrel of oil is taken from the average closing price of West Texas Intermediate (WTI) crude oil in 2015, which is $48.66 per barrel. Value added data comes from OECD Input-Output Tables (2018). We consider three definitions of the goods sector, with both the numerator (value of energy) and the denominator (value added) computed for the same sector definition, either: (i) the manufacturing sector, (ii) manufacturing plus agriculture and construction, and (iii) manufacturing, agriculture, construction, wholesale, retail, and transportation. The values of α that we obtain are, respectively, 0.85, 0.79, and 0.84. Our preferred value is 0.85, very close to two of these three.

For the energy supply elasticities, ϵ_S and ϵ_S^*, we use data from Asker, Collard-Wexler, and De Loecker (2018) on the distribution across oil fields of extraction costs. The data come in the form of quantiles (q = 0.05, 0.10, ..., 0.95), separately for the EU, the US, OPEC, and ROW (q% of oil in the US is extracted at a cost below $\$a$ per barrel, for example). We approximate OECD countries by aggregating the EU and US while for the non-OECD region we aggregate OPEC and ROW. To aggregate the quantiles for two regions, we

[42] We think of value added as the closest proxy to labor cost in the model, since we interpret labor in the model as labor equipped with capital.

combine them, sort the combination by the cost level, and reassemble after taking account of total oil extraction for each region (available from the IEA). The data are plotted on log scales in Figures 6 and 7, to reveal the supply elasticity as the slope.

Figure 6: Calibration of the Extraction Supply Elasticity in Home



The most costly oil fields in either region would be the first to be abandoned under a carbon policy. Thus, the upper end of the cost distribution is the most relevant for calibrating the supply elasticities. Our baseline values of $\epsilon_S = 0.5$ and $\epsilon_S^* = 0.5$ are close to the slope shown in the figures when we consider only costs above the median. Our alternative value of $\epsilon_S^* = 1$ is closer to the slope if we were to use the upper 75% of costs or even all the data.

Lacking this distributional data for coal and natural gas fields, we assume that the distribution for oil extraction is representative of all fossil fuels.

Figure 7: Calibration of the Extraction Supply Elasticity in Foreign