4 Trade in goods and trade in services*

Jonathan Eaton and Samuel Kortum

1. Introduction

Structural gravity modeling has advanced substantially in the last two decades. Trade in merchandise, particularly in manufactures, has either explicitly or implicitly inspired most modeling approaches. In fact, manufactures constitute the largest component of trade, but, according to data reported to the Organization for Economic Cooperation and Development (OECD), trade in services has grown enormously in the last several decades, to the point where it now constitutes about a quarter of total trade involving OECD countries. Our goal in this chapter is to examine basic features of services trade and to ask how well current modeling strategies capture these features. We then propose and quantify extensions to a basic structural gravity model that we think incorporate these features. Our extended model allows us to handle goods trade and services trade in an encompassing framework.

Modeling such trade is daunting because traded services include such diverse activities as tourism, financial services, wholesale and retail trade, innovation, and artistic creation. In an attempt to systematize thinking, the General Agreement on Trade in Services (GATS) of the World Trade Organization (WTO) classifies services exports into four modes of supply:

Mode 1 constitutes a cross-border export. The service is provided by resources located in the exporting country and delivered to a consumer in the importing country. An example would be technical help provided by a customer service representative in India to a U.S. household whose computer has been infected by a virus.

Under mode 2 a person from the importing country travels to the exporting country to consume the service. An example is U.S. college students on spring break traveling to Cancun to buy margaritas at a Mexican bar.

Under mode 3 the exporter provides a commercial presence in the importer. An example is German technology brought to the BMW plant in Spartanburg, South Carolina.

Under mode 4 (the opposite of mode 2), a natural person from the exporter travels to the location of the consumer to provide the service. An example is the Rolling Stones visiting the United States on a concert tour.1
These different modes have different implications for introducing services trade into a general equilibrium economic model. Much services trade under modes 1, 2, and 4 would appear to resemble trade in merchandise in two respects. For one thing, the consumption of the service by the importer involves the recent or contemporaneous employment of factors in the exporter (e.g., the Mexican bartender serving a drink to the inebriated U.S. student). For another, the exports are rival (only one student can sip the margarita at a time). Under mode 3, however, the consumption of the service could occur much later than its production. The research and development (R&D) investment behind BMW’s engineering technology may have occurred years ago. Moreover, BMW could use the same technology in its plant in Germany or in South Africa.

To capture these distinctions we follow Hill (1999) in classifying services exports into two categories, tangible and intangible. Tangible services exports are produced in the exporting country in the same period in which they are consumed in the importing country and they are rival. Intangible services exports could have been produced in the origin long before their use in the destination and are nonrival.

An example illustrating both of these distinctions is that viewers in nearly 200 destinations can simultaneously watch *The Big Sleep* on Netflix, enjoying the efforts of Humphrey Bogart, Lauren Bacall, Howard Hawks, and William Faulkner from seven decades ago, but generating export revenues for Netflix and Warner Brothers even now. We treat the streaming from the Netflix library as a tangible service export. We treat the rights to the *The Big Sleep* as an intangible asset created by Warner Brothers in 1946. It constitutes an intangible asset that Netflix uses for its tangible services export.

Reinsdorf and Slaughter (2009) and Robbins (2009) discuss the thorny accounting issues that trade in intangibles raises. Until recently, producing intangibles has been treated as an intermediate expense for investors, but in 2013 the U.S. Bureau of Economic Analysis (BEA) began treating research and development spending by firms as equivalent to their investment spending on tangible capital assets.2 As with physical capital, the effort involved in creating intangible assets represents current economic activity (e.g., composing a rock song is like building a house), whether it’s counted as investment or intermediate production, but earning income from the asset may involve little or no current resources (earning royalties when the rock song is played on the radio or renting out the house). One distinction between capital and intangible assets, however, is that physical capital is typically rival while intangible assets are not. In contrast to our *Big Sleep* example, if Lufthansa rents a Boeing 777 from Ireland’s GE Capital Aviation Services, that airplane is not available for Air Canada. Another distinction in the national accounts is that, while returns to a country’s physical and intangible capital used abroad contribute to its GNP, they are part of its GDP only for intangible assets or physical capital that is “moveable,” e.g., a French-owned aircraft in Ireland but not a U.K. family’s vacation apartment in Torremolinos.

Of course, how we model trade in services relates to how we model the production and consumption of services generally. How to treat tangible services
doesn’t seem to generate controversy. They’re like merchandise. But researchers have taken alternative approaches to intangibles. Corrado et al. (2009), in particular, propose a (closed economy) accounting framework in which intangibles receive the same treatment as physical capital in three respects: (1) investment (whether construction of capital or creation of intangibles) constitutes a contemporaneous contribution to GDP on the production side; (2) earnings on the assets (whether tangible or intangible) constitute a contemporaneous contribution to GDP on the income side; and (3) the accumulation of assets (whether tangible or intangible) constitute a “source of growth” based on the change in the stock of the asset and its share in production.

While the approach we take to intangible assets here is consistent with the first two characteristics, with the third it is not. As Corrado et al. (2009) point out, for the accumulation of intangibles to constitute a source of growth requires that they have a share in the production function just like physical capital, implying rivalry. Hence their respecification of the accounts to include the accumulation of intangible capital as a source of growth substantially reduces the “Solow residual” in growth accounting. In contrast, our approach, in treating intangibles as non-rival, considers the accumulation of intangibles as the source of this residual. In turning to trade in intangibles, McGrattan and Prescott (2010) and Ramondo (2014) follow Corrado et al. (2009) in treating intangibles as rival, in contrast to our approach here.

We begin with an exploration of data on services trade, documenting its growing, and differential, importance across countries. In particular, services exports now constitute a third or more of the total exports of goods and services of the United Kingdom and United States, but less than 20 per cent for Germany and Japan. In line with previous studies, we find that a standard gravity formulation with exporter–importer fixed effects captures bilateral trade both in services overall and in eight categories of services, nearly as well as trade in goods, with similar distance elasticities.

We then develop a model of trade in goods and services, where we divide services into a tangible and intangible component. We treat tangible services and merchandise similarly. Absorption is related to current production, and output is rival. We model the output of the intangible services sector as nonrival intangible assets that provide technologies for the future production of goods and tangible services. For producers of tangibles to be able to compensate the original creators of their technology requires that they charge a markup over the cost of tangible inputs. Our market structure is consequently imperfectly competitive. Markups on tangibles thus serve as the source of revenue for the creators of intangible assets.

We implement the model numerically to explore its implications for trade in manufactures, tangible services, and intangible assets. The numerical model illustrates, for example, how greater diffusion can benefit all countries, even though it can have negative implications for real wages in some countries.

The next section reviews some basic facts about services trade. In Section 3 we present our model of trade in goods and in services. Section 4 presents some quantitative implications of the model. Section 5 concludes the chapter.
2. Basic facts

According to OECD data, goods continue to dominate services in international trade, but trade in services is growing and for some countries now constitutes a major source of export revenue. Table 4.1 reports services exports (as a share of total exports) and services imports (as a share of total imports) for 20 OECD countries for 1985, 2000, and 2015. Only for Luxembourg in 2000 and in 2015 does services constitute the majority of trade, but the share of services in trade grew in all but a handful of cases. For some large economies, such as the United Kingdom and United States, services exports represent more than a third of the total. But for others, such as Japan and Germany, the services share is less than 20 per cent.

Services trade comprises a wide ranging set of activities. To get a more detailed breakdown of services trade we turn to the World Input-Output Database (WIOD). The WIOD reports annual amounts of production, absorption, and bilateral trade among 43 countries, partitioning economic activity into around 50 sectors. Table 4.2 provides a list of the different subcategories of services in

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<td>United States</td>
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1 Source: OECD National Accounts Data (all OECD countries with complete data as of 1985).
2 German data before 1991 are estimated based on today’s boundaries.
3 Total trade (exports or imports) designates goods and services.
### Table 4.2 Industry correspondence

<table>
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<tr>
<th>Category</th>
<th>Industry description</th>
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<td>Water collection, treatment and supply</td>
</tr>
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<td>Omitted</td>
<td>Sewerage; waste collection, treatment and disposal activities; materials recovery; remediation activities and other waste management services</td>
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<td>Construction</td>
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<td>Wholesale Retail</td>
<td>Wholesale trade, except of motor vehicles and motorcycles</td>
</tr>
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<td>Transportation</td>
<td>Postal and courier activities</td>
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<td>Accommodation and food service activities</td>
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<td>Publishing activities</td>
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<tr>
<td>Communication</td>
<td>Telecommunications</td>
</tr>
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<td>Insurance, reinsurance and pension funding, except compulsory social security</td>
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<td>Activities auxiliary to financial services and insurance activities</td>
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<tr>
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<td>Real estate activities</td>
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<td>Professional</td>
<td>Legal and accounting activities; activities of head offices; management consultancy activities</td>
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<td>Architectural and engineering activities; technical testing and analysis</td>
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<td>Scientific research and development</td>
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<td>Advertising and market research</td>
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<td>Other professional, scientific and technical activities; veterinary activities</td>
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<td>Administration</td>
<td>Administrative and support service activities</td>
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<td>Other</td>
<td>Public administration and defence; compulsory social security</td>
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<td>Other</td>
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<td>Human health and social work activities</td>
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<tr>
<td>Other</td>
<td>Other service activities</td>
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<tr>
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<td>Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use</td>
</tr>
<tr>
<td>Omitted</td>
<td>Activities of extraterritorial organizations and bodies</td>
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</table>
the WIOD. For the purposes of our analysis here we aggregate them into eight categories indicated on the first column of the table (with some codes omitted from our categorization).

Table 4.3 provides a breakdown of total services exports into the eight categories for 43 countries in 2010 based on the WIOD. They demonstrate some striking patterns of specialization. For France, 37.6 per cent of services exports are in wholesale and retail trade, while communications constitute 39.6 per cent of India’s services exports. While call centers may explain India’s revealed comparative advantage in communications, other patterns may reflect differences in how different countries classify different activities.7

To what extent does services trade resemble trade in merchandise, for which data have been available much more comprehensively? Let’s first gauge the extent to which services are traded across countries compared with merchandise and manufactures. Based on the WIOD for 2010, we calculate, for each sector, the sum of what’s traded between countries relative to total world production, which equals total world absorption. Specifically, denoting sales in sector \( j \) to destination \( n \) from source \( i \) as \( X_{ni}^{j} \), we calculate:

\[
O_{ni}^{j} = \frac{\sum_{i,n \neq i}X_{ni}^{j}}{\sum_{i,n}X_{ni}^{j}}.
\]

The bottom row of Table 4.4a, labeled “off diagonal ratio,” reports the results (repeated in Tables 4.4b and 4.4c for convenience). For goods, what’s internationally traded is 21 per cent of total production and the traded share for manufactures is slightly higher. In contrast, the share for total services is only 3 per cent. Only for transportation is the traded share even one third of what it is for all goods or for manufactures.

To the extent that services are traded internationally, do geographic barriers such as distance play the same role in services trade as in merchandise trade? For decades the standard gravity model has been the workhorse tool for describing bilateral trade patterns. We use WIOD data from 2010 to look at patterns of bilateral trade in all goods, manufactures, services, and our eight categories of services.8 We relate destination \( n \)’s imports from source \( i \) in sector \( j \), \( X_{ni}^{j} \), \( n \neq i \), to a fixed effect \( S_{ij}^{j} \) for sector \( j \) in source \( i \), to a fixed effect \( D_{nj}^{j} \) for sector \( j \) in destination \( n \), and to characteristics connecting origin \( i \) and destination \( n \). Our bilateral indicators indexed by \( k, t_{ni}^{k} \), are the log of the distance between \( n \) and \( i \) and fixed effects for countries \( n \) and \( i \) sharing a common language, common border, and former colonial connection, all taken from the CEPII website. We estimate three versions of a gravity specification found in the literature.

Table 4.4a reports the results of estimating the equation:

\[
\ln(X_{ni}^{j}) = S_{ij}^{j} + D_{nj}^{j} + \sum_{k} \gamma_{k}^{j} t_{ni}^{k} + \epsilon_{ni}^{j} \tag{1}
\]
Table 4.3 Services export shares

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<th>Country</th>
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<th>Wholesale retail</th>
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2 Export share is taken as a share of total export of services.
## Table 4.4a Gravity model estimates (ordinary least squares)

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<th>Retail</th>
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The dependent variable is the logarithm of a destination country’s imports from an origin country. 

* p < 0.05
** p < 0.01
*** p < 0.001
Table 4.4b Gravity model estimates (Poisson pseudo maximum likelihood)

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The dependent variable is the level of a destination country’s imports from an origin country. T statistics in parentheses.

* p < 0.05
** p < 0.01
*** p < 0.001
### Table 4.4c: Gravity model estimates (multinomial pseudo maximum likelihood)

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<td>-0.927</td>
<td>-2.839</td>
<td>-7.112***</td>
</tr>
<tr>
<td></td>
<td>(-9.02)</td>
<td>(-8.63)</td>
<td>(-0.12)</td>
<td>(-0.87)</td>
<td>(-1.84)</td>
<td>(-2.58)</td>
<td>(-6.07)</td>
<td>(-0.95)</td>
<td>(-0.79)</td>
<td>(-1.61)</td>
<td>(-3.44)</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>0.00752</td>
<td>0.00752</td>
<td>0.0044</td>
<td>0.0051</td>
<td>0.086</td>
<td>0.042</td>
<td>0.070</td>
<td>0.001</td>
<td>0.0032</td>
<td>0.0471</td>
<td>0.111</td>
</tr>
</tbody>
</table>

The dependent variable is the logarithm of a destination country’s imports from an origin country.

* t statistics in parentheses.

* p < 0.05
** p < 0.01
*** p < 0.001
by ordinary least squares (OLS). Here $\gamma^j_k$ is the coefficient on the relevant bilateral indicator $i^k_n$ and $\varepsilon^j_{ni}$ is an error term reflecting idiosyncratic components of exports from source $i$ to destination $n$ in sector $j$. With 43 countries there are a total of 1806 ($= 43^2 - 43$) observations of bilateral trade. For several of the services categories we have had to drop observations for which $X^j_{ni} = 0$, reducing the number of observations accordingly (as reported in the row labeled “N”).

Note first that the coefficients on distance are very similar for all goods, manufactures, and services. Trade decays with distance with an elasticity around 1.33 to 1.38. Somewhat surprisingly, services appear slightly less sensitive to the trading partners’ sharing a common border or language.

Note also that, with the exception of wholesale and retail trade, gravity is less robust for services. The $R^2$’s for goods and manufactures are solidly above 0.9 but with services dip as low as 0.73. But for services as a whole the $R^2$ is 0.875. The overall picture is that gravity plays nearly as strong a role in services trade as it does in goods trade.

To avoid the loss of information in removing observations of zero trade in OLS estimation, Santos Silva and Tenreyro (2006) propose estimating the gravity equation:

$$X^j_{ni} = \exp \left( S^j_i + D_n^j + \sum_k \gamma^j_k i^k_n \right) + \varepsilon^j_{ni}$$

by Poisson pseudo-maximum likelihood (PPML). Table 4.4b reports the results of applying this estimation strategy to our data. Some observations still have to be dropped (in increments of 42) if a country doesn’t report any exports or any imports in some category (which prevents estimation of the corresponding exporter or importer fixed effect).

Eaton et al. (2013), henceforth EKS, derive a theory of zeros in the trade data due to a finite number of firms. They propose estimating the gravity equation:

$$\frac{X^j_{ni}}{X^j_{n}} = \exp \left( S^j_i + D_n^j + \sum_k \gamma^j_k i^k_n \right) + \varepsilon^j_{ni}$$

by multinomial pseudo-maximum likelihood (MPML). Table 4.4c presents the results of applying this estimation strategy to our data.

In almost all sectors the distance elasticities are lower using PPML and somewhere in between using MPML. Otherwise the results are very similar.

Despite the lower overall tradability of services, as we saw in Table 4.1, services account for a large share of exports from certain countries (related to the large share of services in their GDP). To explore relative specialization in services versus manufactures Figures 4.1 to 4.3 report the exporter fixed effects from our gravity regressions for the 43 countries in our sample (with the relevant
three-letter country code indicated in Table 4.3). Specifically, for each country \(i\) we calculate:

\[
\Psi_j^i = \frac{\exp(S_j^i)}{\sum_{j=1}^{N} \exp(S_j^i)},
\]

for \(j\) corresponding to services (\(S\)) and to manufactures (\(M\)), using the results from OLS estimation (Table 4.4a). To accommodate the vast size differences across countries we organize the countries in descending order of their gravity exporter effects \(\Psi_j^S\) into three different charts. Note the different scales across the three figures.

Figure 4.1 reveals a striking degree of specialization across the largest exporters. The United States, United Kingdom, and Canada contribute much more to services exports while China and Germany are heavily skewed toward manufactures. The remaining figures reveal similarly strong patterns of specialization for smaller countries.

To summarize, while services are much less traded than goods or manufactures, gravity provides a good statistical description. Exporter fixed effects reflect surprising degrees of specialization. We now turn to a model designed to capture these patterns.

3. A model of trade in goods and services

We build on Eaton and Kortum (1999), henceforth EK (1999); Eaton and Kortum (2001), henceforth EK (2001); Eaton and Kortum (2002), henceforth EK (2002); Bernard et al. (2003), henceforth BEJK (2003); Eaton and Kortum (2007),
Figure 4.2 Gravity exporter effects (top 6–25 for services)
Figure 4.3 Gravity exporter effects (bottom 18 for services)
henceforth EK (2007); Ramondo and Rodriguez-Clare (2013), henceforth RR-C (2013); and Arkolakis et al. (2014), henceforth ARR-CY (2014). As in EK (1999) and EK (2007) intangible ideas emerge from a process of invention. These ideas, each associated with a potential production technology, diffuse over time across countries, with the original inventors earning royalties from the use of their ideas around the world. These royalties constitute earnings on intangibles, a component of services exports. EK (1999) ignored international trade, so that the correlation of productivities around the world implied by the use of the same technology in different countries was irrelevant. EK (2007) assumed that diffusion led to perfect correlation of efficiency across borders, which made their model difficult to apply to multicountry data. RR-C (2013) and ARR-CY (2014) assumed instantaneous diffusion, but allow for positive but imperfect correlation.

Our model of production and trade reflects the fact that services can sometimes take the form of a rival service currently provided by inputs in the selling country to consumers in the buying country. It may also reflect the return on nonrival intangible assets produced in the past by what we will call the “intangibles sector.”

We consider an arbitrary integer number \( N \) of countries, indexed by \( i \) (as innovators), by \( l \) (as producers), and by \( n \) (as destinations). Each country has three sectors, (tangible) manufactures \( M \), (tangible) services \( S \), and intangibles \( I \). In each tangible sector there are a unit continuum of varieties. The output of each tangible sector is a constant elasticity of substitution (CES) aggregate over its varieties, with elasticity of substitution \( \sigma^j, j \in \{M, S\} \). Final absorption consists entirely of tangibles, with manufactures having a Cobb-Douglas share \( \alpha^M \) and services a share \( \alpha^S = 1 - \alpha^M \) in preferences.\(^9\) The intangibles sector is the source of new production technologies, which we turn to next.

### 3.1. Technologies

Each sector produces output by combining, in a Cobb-Douglas production function, three rival inputs: labor and intermediates from each of the two tangible sectors. For sector \( j \in \{M, S, I\} \), we denote the output elasticity of the sector \( j \) intermediate by \( \beta^{jL}, j \in \{M, S\} \). Due to constant returns to scale, the output elasticity of labor is:

\[
\beta^{jL} = 1 - \beta^{jM} - \beta^{jS}.
\]

Production of each variety in each tangible sector also requires a non-rival input, which we call the production technology. While we treat the output elasticities of rival inputs as common across the continuum of varieties within a tangible sector, the production technology varies across varieties and countries.

A production technology is the output, at some date in the past, of the intangible sector in some country \( i \), which we assume maintains property rights over it. We distinguish a technology by this origin country, by the sector and
variety to which it applies, and by whether it’s exclusively available in country \( i \) or has diffused everywhere.\(^{10} \) We can ignore the age of a technology, since its value is based on its productive efficiency relative to other available technologies. As this description makes clear, a technology is an intangible asset and, as we describe later, producers will pay for the right to use one.\(^ {11} \)

Consider first technologies for producing varieties in tangible sector \( j \) that were developed in country \( i \) and remain exclusive to it, so that \( i \) is the only possible producer (i.e., \( l = i \)). These intangibles do not generate services exports for country \( i \) since the technology can only be used domestically.

As we show in appendix equation (27), our assumptions about the stochastic arrival of ideas imply that the number of such technologies to produce a variety with efficiency above \( z \) is distributed Poisson with parameter:

\[
L_i^E(z) = T_i^E z^{-\theta_i}. \tag{4}
\]

Here the parameter \( T_i^E \) reflects the size of country \( i \)'s pool of exclusive technology in sector \( j \). The probability distribution of country \( i \)'s most efficient exclusive technology for a variety in sector \( j \), delivering output per bundle of inputs \( Z_j^E \), is thus:

\[
Pr[Z_j^E \leq z] = \exp(-T_i^E z^{-\theta_i}), \tag{5}
\]

which is the Poisson probability that no technology better than \( z \) is available. Realizations are independent across source countries \( i \) for these exclusive technologies.

We now turn to technologies from country \( i \) that have diffused everywhere. Since these technologies may be used by producers in other countries, these intangibles can generate services exports for country \( i \).

A diffused technology delivers a different efficiency in each potential production location \( l \), but we allow for that efficiency to be correlated across countries. Furthermore, following RR-C (2013) and Ramondo (2014), we assume that diffusion of a technology from an origin country \( i \) to a producer in location \( l \) diminishes its productivity there by a factor \( h_{jl}^i \geq 1 \), where we normalize \( h_{ji}^i = 1 \). These parameters, which we call iceberg transfer costs (for technology), moderate the level of intangible services exports much like iceberg transport costs (introduced in the next section) moderate the level of goods and tangible services exports.

As we show in appendix equation (28), our assumptions about the diffusion of ideas imply that the number of technologies diffused from \( i \) for producing a variety with efficiency above \( z_l \) in at least one location \( l \) is distributed Poisson with parameter:

\[
L_i^D(z_1, z_2, \ldots, z_N) = T_i^D \left( \sum_{l=1}^{N} (z_l h_{jl}^i)^{-\theta_l/(1-\rho^l)} \right)^{1-\rho^l}. \tag{6}
\]

Here \( T_i^D \) reflects the size of country \( i \)'s pool of diffused ideas in sector \( j \) and \( \rho^l \in [0, 1) \) reflects the correlation across locations of the efficiency delivered by a diffused technology.
Consider the most efficient such technology from the perspective of a producer of a variety in a given location \( l \), delivering output per bundle of inputs \( Z^{D}_{ji} \). The joint distribution of the \( Z^{D}_{ji} \) across all potential production locations is:

\[
\Pr[Z^{D}_{ji} \leq z_1, Z^{D}_{ji} \leq z_2, \ldots, Z^{D}_{Ni} \leq z_N]
= \exp \left( -T^{D}_{i} \left( \sum_{l=1}^{N} (z_l h^l_i)^{-\beta/(1-\rho)} \right)^{1-\rho} \right),
\]

which is the Poisson probability that no technology better than \( z_l \) for any location \( l \) is available. Realizations are still independent across source countries \( i \), but are correlated across production locations \( l \), with the correlation increasing in \( \rho^l \). For a given set of \( \{z_l\}_{l=1}^{N} \), the probability (7) is increasing in the technology transfer costs, since a larger \( h^l_i \) reduces the likelihood that a technology from \( i \) will yield high efficiency when used in any location \( l \neq i \).

### 3.2. Costs

We denote the price index of tangible sector \( j \) output in country \( l \) as \( p^j_l \), \( j \in \{M, S\} \). With a wage \( w_i \), the cost of a bundle of inputs to produce sector \( j' \in \{M, S, I\} \) output in production location \( l \) is thus:

\[
b^j_l = (p^M_l)^{h^M_j} \left( p^S_l \right)^{h^S_j} (w_i)^{h^I_j}.
\]

Later, we will connect the price indices \( p^j_l \) to these input bundle costs, technologies, and market structure. For now, we simply take them as given to focus on unit production costs in locations \( l \), and costs of delivery to a destination \( n \), which inherit a distribution from the technologies underlying these costs.

The unit cost of producing a variety of tangible sector \( j \) in country \( l \) is \( b^j_l / Z^I_l \), where \( Z^I_l \) is the random level of efficiency of the best available technology. We posit an iceberg transport cost \( d^j_{ln} \geq 1 \) to deliver a unit to \( n \) from \( l \). The unit cost of producing a variety of tangible sector \( j \) in country \( l \) after delivery to destination \( n \) is then:

\[
C^j_{ln} = \frac{b^j_l d^j_{ln}}{Z^I_l}.
\]

For location \( l \) to be able to provide the variety at a cost \( C^j_{nl} \leq c \) thus requires that its efficiency satisfies:

\[
Z^I_l \geq \frac{b^j_l d^j_{nl}}{c}.
\]
Applying equation (4), the number of exclusive technologies from $I$ that exceed this threshold is distributed Poisson with parameter

$$\lambda_I^{E}(\frac{b'_1 d_{ml}}{c}) = T_I^{E}(\frac{b'_1}{d_{ml}})^{-\theta_1} c^{\theta_1}. \quad (9)$$

For a diffused technology that originated in country $i$ to serve destination $n$ at a cost below $c$ its efficiency in at least one production location $l$ must also satisfy (8). From (6), the number of such technologies is distributed Poisson with parameter:

$$\lambda_I^{D}(\frac{b'_1 d_{ml}}{c}, \frac{b'_2 d_{m2}}{c}, ..., \frac{b'_N d_{mN}}{c}) = T_I^{D}\left(\sum_{l=1}^{N} (b'_l d_{ml} h'_l)^{-\theta_l/(1-\rho_l)}\right)^{1-\rho_l} c^{\theta_1}. \quad (10)$$

Since technologies from different countries, and those that are exclusive and diffused, are all independent of each other, the number of technologies that can provide a sector $j$ variety to destination $n$ at unit cost below $c$, regardless of source or diffusion status, is distributed Poisson with parameter:

$$\lambda_n^{j}(c) = \sum_{l=1}^{N} \lambda_I^{E}(\frac{b'_1 d_{ml}}{c}) + \sum_{l=1}^{N} \lambda_I^{D}(\frac{b'_1 d_{ml}}{c}, \frac{b'_2 d_{m2}}{c}, ..., \frac{b'_N d_{mN}}{c}) = \Phi_n^{j} c^{\theta_1} \quad (11)$$

where:

$$\Phi_n^{j} = \sum_{l=1}^{N} T_I^{E}(\frac{b'_1 d_{ml}}{c})^{\theta_1} + \sum_{l=1}^{N} T_I^{D}\left(\sum_{l=1}^{N} (b'_l d_{ml} h'_l)^{-\theta_l/(1-\rho_l)}\right)^{1-\rho_l}. \quad (12)$$

We can now derive the distribution of the lowest cost of a tangible sector $j$ variety available in country $n$ (produced in any location $l$ using technologies from any source $i$). The probability that all costs are above $c$ (hence the lowest as well) is the probability that no cost is below $c$. The distribution of this lowest cost $C_n^{j}$ is:

$$G_n^{j}(c) = \Pr[C_n^{j} \leq c] = 1 - \exp(-\Phi_n^{j} c^{\theta_1}), \quad (13)$$

which is 1 minus the Poisson probability that no cost is below $c$. The distribution of costs in $n$ depends, through $\Phi_n^{j}$, on all iceberg transport costs into $n$ and on all iceberg transfer costs between every pair of countries.

### 3.3. Sources of production and technology

We now turn to which location $l$ can serve market $n$ at the lowest cost with a technology originating from $i$. We start with the probability that a technology originating from $i$ supplies $n$ at the lowest cost.

First consider technology that’s exclusive to $i$, so that $l = i$. The probability that such a technology is lowest cost in supplying $n$ is the ratio of the Poisson parameter (9) to the Poisson parameter (11):

$$n_n^{lE} = \frac{T_I^{E}(b'_1 d_{ml})^{-\theta_1}}{\Phi_n^{j}}. \quad (14)$$
Note how \( c \) cancels. This share is like that in EK (2002), as country \( i \)'s share depends on its pool of exclusive technology downweighted by its cost of inputs and the iceberg transport cost of delivering tangible goods to country \( n \). The response of the share to these costs is governed by \( \theta^j \). The new piece is in the denominator (12), which includes terms capturing the possibility that \( n \) is supplied using a technology that is not exclusive to any country.

For technology that’s diffused from \( i \), the probability that such a technology is lowest cost in supplying \( n \) (from any location \( l \)) is the ratio of the Poisson parameter (10) to the Poisson parameter (11):

\[
\pi_{nli}^D = \frac{T_i^D \left( \sum_{j=1}^{N} (b_j d_{il}^l h_{i(l)})^{-\theta^j/(1-\rho^j)} \right)^{1-\rho^j}}{\phi_n^j},
\]

where the dot in the subscript indicates that the production location \( l \) could be anywhere. Here, country \( i \)'s share (in supplying the technology) depends on its pool of diffused technology downweighted by the iceberg transfer cost of getting the technology from \( i \) to any of the potential producing countries \( l \) together with the cost of inputs in \( l \) and the iceberg transport cost of shipping goods from \( l \) to \( n \).

As we show in appendix equation (30), the probability that country \( l \) is the supplier, when a technology that diffused from \( i \) is used to supply \( n \), is:

\[
\pi_{nlj}^D = \frac{(b_j d_{jl} h_{l})^{-\theta^j/(1-\rho^j)}}{\sum_{j=1}^{N} (b_j d_{jl} h_{l})^{-\theta^j/(1-\rho^j)}},
\]

where the vertical bar in the subscript indicates conditioning on a diffused technology from \( i \) being used to supply \( n \). In this expression, since each producing country has access to the same technologies that diffuse from \( i \), the shares do not depend on pools of technology. Instead \( l \)'s share depends on its access to \( i \)'s diffused technology (as governed by the iceberg transfer cost), its cost of inputs, and its access to country \( n \) (as governed by the iceberg transport cost).

The response of this share to these costs is governed by \( \theta^j/(1 - \rho^j) \), which is increasing in the correlation parameter \( \rho^j \). If the efficiency in different production locations of the same diffused technology is quite similar (\( \rho^j \) near 1), slight differences in input or transport costs across locations will have a large effect on which location ends up producing with such technologies.

The probability that country \( l \) supplies \( n \) using a technology that diffused from \( i \) is the product of the probability that such a technology provides the lowest cost in \( n \) and the probability that country \( l \) is the lowest cost location for using it, when serving destination \( n \):

\[
\pi_{nl}^D = \pi_{nli}^D \cdot \pi_{nlj}^D
\]

where

\[
\pi_{nl}^D = \frac{T_i^D \left( \sum_{j=1}^{N} (b_j d_{il}^l h_{i(l)})^{-\theta^j/(1-\rho^j)} \right)^{1-\rho^j}}{\phi_n^j} \cdot \frac{(b_j d_{jl} h_{l})^{-\theta^j/(1-\rho^j)}}{\sum_{j=1}^{N} (b_j d_{jl} h_{l})^{-\theta^j/(1-\rho^j)}}.
\]
Combining (15) and (14), the probability that a variety purchased in \( n \) is produced in \( l \) using an idea from \( i \) is:

\[
\pi_{nli} = \delta_{li} \pi_{ni}^E + \pi_{nli}^D
\]

where \( \delta_{li} = 0 \) if \( l \neq i \) and \( \delta_{li} = 1 \) if \( l = i \). We now have a full description of how technology from different sources is used in different production locations to serve different markets. Since country \( n \) is necessarily supplied (with a variety of tangible sector \( j \)) by some country \( l \) using a technology from some country \( i \), the probabilities satisfy:

\[
\sum_{i=1}^{N} \sum_{l=1}^{N} \pi_{nli} = 1.
\]

### 3.4. Market structure and markups

We treat the difference between revenues and costs in using a technology as a return to the creator of the technology, which we refer to as a royalty. Deriving the royalty share of revenues requires our characterizing the distribution of the markups, which we turn to now.

We assume that potential producers of a tangible variety engage in Bertrand competition in each market \( n \) where they sell, regardless of whether the buyer is another producer or a household. A result of this competition is that the low-cost producer of a variety serves the market and its price equals either the cost of the second lowest-cost potential supplier of that variety to market \( n \) or the monopoly price, whichever is lower.

This market structure leads to random markups \( \bar{M}_n^j \) for a variety of tangible sector \( j \) supplied to country \( n \). As we show in appendix equation (31), the distribution of these markups takes the very convenient form of a truncated Pareto distribution:

\[
\Pr[\bar{M}_n^j \leq m] = \begin{cases} 
1 - m^{-\sigma^j} & 1 \leq m < \bar{m}^j \\
1 & m = \bar{m}^j
\end{cases},
\]

where the truncation point is the monopoly markup:

\[
\bar{m}^j = \frac{\sigma^j}{\sigma^j - 1}.
\]

This distribution of markups, together with the distribution of costs (13), determines the distribution of tangible sector \( j \) prices in country \( n \).

### 3.5. The price index

Until this point, our derivations applied to a particular variety (such as the probability \( \pi_{nli}^j \) that a variety of tangible sector \( j \) is supplied to \( n \) by \( l \) using a
technology from $i$). In deriving the price index, however, we need to integrate across varieties, so it’s convenient to give them an index. Let $P^j_n(\omega)$ denote the price in country $n$ of variety $\omega$ in tangible sector $j$. The price index is thus:

$$p^j_n = \left( \int_0^1 [P^j_n(\omega)]^{-\sigma_j} d\omega \right)^{1/(1-\sigma_j)}.$$

By treating the individual prices as random variables, as we show in appendix equation (34), we obtain an expression connecting this price index to the parameter of the cost distribution (12):

$$p^j_n = \gamma^j \Phi^j_n^{-1/\theta_j}, \quad (17)$$

where:

$$\gamma^j = \Gamma \left( \frac{2\theta_j - (\sigma_j - 1)}{\theta_j} \right)^{1/(1-\sigma_j)} \left( 1 + \frac{\sigma_j - 1}{\theta_j - (\sigma_j - 1)} (\bar{m}^j)^{-\theta_j} \right)^{1/(1-\sigma_j)}.$$

The CES parameter $\sigma^j$ matters (in this model) only through the constant $\gamma^j$.

### 3.6. Royalties

Tangible goods are sold in country $n$ at a markup over the cost of the inputs used to produce them (including any transport cost to deliver them). We interpret the resulting wedge between revenue and tangible cost as the value of the intangible services embodied in country $n$’s absorption. Ultimately, this value of intangible services will flow in the form of royalties to the country whose intangible sector generated the intangible assets.

As we show in appendix equation (35), the value of intangible services $X^{j,j}_n$ embodied in country $n$’s absorption of tangible sector $j$ goods is:

$$X^{j,j}_n = \frac{1}{1 + \theta_j} X^j_n.$$

We return to the trade shares to track how the payments for these intangible services flow as royalties earned by the owners of the intangible assets that generate them.

Since there are a continuum of varieties in each tangible sector, the probability $\pi^j_{ni}$ that a variety consumed in $n$ was produced in $l$ using a technology from $i$ is also the share of spending on such varieties in $n$. An implication is that the share of $l$’s production in $n$’s sector $j$ absorption (without regard to the source of the technology $i$) is:

$$\pi^{j,T}_{ni} = \sum_{i=1}^N \pi^{j}_{ni}, \quad (18)$$
where the $T$ superscript indicates that this term is the share of $n$’s spending on tangible sector $j$ goods imported from country $l$.

The share of origin $i$’s technologies used in $n$’s sector $j$ absorption (without regard to the production location $l$) is:

$$\pi^i_{nj} = \sum_{l=1}^{N} \pi^i_{nlj},$$

(19)

where the $I$ superscript indicates that this term is the share of $n$’s absorption of intangible services (embodied in $n$’s spending on tangible sector $j$) ultimately paid as royalties to country $i$.

The royalties earned by country $i$’s sector $j$ technologies used for production in $l$ and absorbed in destination $n$ is:

$$X^i_{nli} = \pi^i_{nlj}X^j_n = \frac{1}{1 + \theta^i} \pi^i_{nlj}X^j_n.$$

We treat royalty income as collected at the point of production rather than at the point of absorption, so that if a producer in $l$ uses a technology from $i$ to supply a consumer in $n$, we count $i$ as exporting the intangible to country $l$ not to $n$. In this case sector $j$ intangible services exports from origin $i$ to location $l$ are:

$$X^i_{li} = \sum_{n=1}^{N} X^j_{nli}.$$

Summing over production locations (including country $i$ itself), royalty income of country $i$ from sector $j$ technologies is:

$$R^i_j = \sum_{l=1}^{N} X^j_{li}.$$

Summing over sectors, country $i$’s total royalty income is:

$$R^i_j = \sum_{j \in \{ M, S \}} R^i_j.$$  

(20)

3.7. Production and absorption

We denote the gross output of sector $j'$ in country $l$ as $Y^j_l$, $j' \in \{ M, S, I \}$. For the two tangible sectors, $j \in \{ M, S \}$:

$$Y^j_l = \sum_{n=1}^{N} \pi^j_{nl}X^j_n.$$
Sector $j$ tangible absorption in country $n$ consists of final demand $X_n^{j,F}$ plus intermediate demand, so that total absorption of tangible sector $j$ by country $n$ is:

$$X_n^j = X_n^{j,F} + \beta^j Y_n^j + \sum_{j' \in \{M,S\}} \beta^{j'} X_n^{j'} + \frac{\theta^j}{1 + \theta^j} Y_n^j.$$

Final demand for sector $j$ output in country $n$ is a share $\alpha^j$ of total final absorption, which itself consists of labor income $w_n L_n$, royalty income $R_n$ from intangibles, less what’s invested in intangibles $Y_n^j$ together with any net exports $N_n$:

$$X_n^{j,F} = \alpha^j (w_n L_n + R_n - Y_n^j - N_n).$$

Labor income is simply labor’s share of total production costs in each sector:

$$w_n L_n = \sum_{j \in \{M,S\}} \beta^j \frac{\theta^j}{1 + \theta^j} Y_n^j + \beta^j Y_n^j.$$

### 3.8. Equilibrium

We can combine these various pieces to form three systems of equations. The first system gives absorption of each country’s tangibles in each sector given final sectoral demand and trade shares:

$$X_n^j = X_n^{j,F} + \beta^j Y_n^j + \sum_{j' \in \{M,S\}} \beta^{j'} \frac{\theta^{j'}}{1 + \theta^{j'}} \sum_{n=1}^{N} \pi_{n,n}' X_n^{j'}.$$  \hfill (22)

The second system gives labor income in terms of sectoral absorption:

$$w_i L_i = \beta^j Y_n^j + \sum_{j \in \{M,S\}} \beta^j \frac{\theta^j}{1 + \theta^j} \sum_{n=1}^{N} \pi_{n,n} X_n^j.$$  \hfill (23)

Incorporating (21) and (20) into (22) gives:

$$X_n^j = \alpha^j (w_i L_i + R_i - Y_n^j - N_i) + \beta^j Y_n^j + \sum_{j' \in \{M,S\}} \sum_{n=1}^{N} \beta^{j'} \frac{\theta^{j'}}{1 + \theta^{j'}} \pi_{n,n}' X_n^{j'}$$

$$= \alpha^j (w_i L_i - Y_n^j - N_i) + \beta^j Y_n^j + \sum_{j' \in \{M,S\}} \sum_{n=1}^{N} \left( \alpha^j \frac{1}{1 + \theta^j} \pi_{n,n}' + \beta^{j'} \frac{\theta^{j'}}{1 + \theta^{j'}} \pi_{n,n}' X_n^{j'} \right).$$  \hfill (24)
Given wages, prices are determined by the third system of equations, for \( j \in \{ M, S \} \) and \( j' \neq j \):

\[
(p_{n}')^{-\theta j} = \gamma' \sum_{i=1}^{N} T_{i}^{jE} \left( w_{i}^{1-\beta j'-\beta} (p_{i}')^{-\beta j'} (p_{i}')^{-\theta j'} d_{i}' \right) \\
+ \gamma' \sum_{i=1}^{N} T_{i}^{jD} \left( \sum_{l=1}^{N} \left( w_{l}^{1-\beta j'-\beta} (p_{l}')^{-\beta j'} (p_{l}')^{-\theta j'} d_{l}' l_{k}' \right) \right)^{1-\rho j}.
\]

(25)

Together, the system of equations (23), (24) and (25) determine wages \( w_{l} \), price indices \( p_{i}' \), and final absorption \( X_{n}' \), \( j \in \{ M, S \} \), given each country’s production of intangibles \( Y_{j}' \), which we take as exogenous in this static analysis.

### 3.9. International income accounts

Having laid out the model, we can now discuss how it maps into an international system of accounts with trade in services and intangibles. Doing so will help to connect the model to the data. Two additional definitions will make this task easier.

First, we define the concept of sectoral value added. For the intangible sector, value added is standard:

\[
y_{l} = Y_{l} - \beta^{M} Y_{l} - \beta^{S} Y_{l} = \beta^{L} Y_{l} = w_{l} L_{l}.
\]

where \( L_{l} \) denotes labor employed in sector \( j \in \{ M, S, I \} \). Since there is no physical capital in the model, and since we have assumed that the intangible sector does not itself use specific technologies to invent new ones, value added reduces to labor income earned in the sector. For the two tangible sectors, \( j \in \{ M, S \} \), we include intangible inputs in value added since they represent a return on intangible assets:

\[
y_{l} = Y_{l} - \beta^{M} \frac{\theta^{j}}{1 + \theta^{j}} Y_{l} - \beta^{S} \frac{\theta^{j}}{1 + \theta^{j}} Y_{l} \\
= \frac{\theta^{j}}{1 + \theta^{j}} \beta^{L} Y_{l} + \frac{1}{1 + \theta^{j}} Y_{l} = w_{l} L_{l} + \frac{1}{1 + \theta^{j}} Y_{l}.
\]

Our treatment of the services of intangibles in value added (for the tangible goods sectors) is parallel to how we would treat physical capital (if it were included in the model) where the value of capital services is part of value added even if the sector rents the capital from elsewhere.

Second, since there is no final demand for intangibles, we define demand for intangibles used in tangible good production in country \( l \) as:

\[
X_{l} = \sum_{j \in \{ M, S \}} \frac{1}{1 + \theta^{j}} Y_{l}.
\]
To complete the accounts for intangibles, we can define bilateral trade in intangibles by:

\[ X_{ih}^l = \sum_{j \in \{M, S\}} X_{ij}^l, \]

Using this expression for bilateral trade, demand for intangibles by country \( l \) satisfies:

\[ X_l^l = \sum_{i=1}^{N} X_{ih}^l, \]

and royalty income of country \( i \) satisfies:

\[ R_i = \sum_{i=1}^{N} X_{ih}^l. \]

### 3.9.1. Income side

We let \( y_i \) denote GDP of a country \( i \). From the income side, GDP is measured as payments to the two factors, wage income to rival labor and royalty income to non-rival technology assets:

\[ y_i = w_i L_i^l + w_i L_i^M + w_i L_i^S + R_i^M + R_i^S \]
\[ = w_i L_i + R_i. \]

### 3.9.2. Production side

From the production side, we can sum sectoral value added of a country \( l \) to get:

\[ y_l^l + y_l^M + y_l^S = w_l L_l^l + w_l L_l^M + \frac{1}{1 + \theta^M} Y_l^M + w_l L_l^S + \frac{1}{1 + \theta^S} Y_l^S \]
\[ = w_l L_l + \frac{1}{1 + \theta^M} Y_l^M + \frac{1}{1 + \theta^S} Y_l^S \]
\[ = w_l L_l + X_l^l. \]

We can therefore express GDP in country \( l \) as:

\[ y_l = y_l^l + y_l^M + y_l^S + N_l^l, \]
where the last term is country $l$'s net exports of intangibles:

$$N_i^l = R_i - X_i^l = \sum_{p \neq i}^N X_{pi}^l - \sum_{i \neq l}^N X_{il}^l.$$

### 3.9.3. Expenditure side

Since we do not model physical investment, spending on investment $I_n$ in country $n$ is equal to its intangible sector output $I_n^i = Y_n^i$. Spending on consumption (broadly interpreted) is final demand for tangible goods:

$$C_n = \sum_{j \in \{M, S\}} X_{nj}^{jF} = \sum_{j \in \{M, S\}} \left( X_{nj}^j - \beta^M \gamma_n^j - \beta^M \frac{\theta^M}{1 + \theta^M} Y_n^M - \beta^S \frac{\theta^S}{1 + \theta^S} Y_n^S \right).$$

Thus, from the spending side:

$$C_n + I_n = \sum_{j \in \{M, S\}} \left( X_{nj}^j - \beta^M \frac{\theta^M}{1 + \theta^M} Y_n^M - \beta^S \frac{\theta^S}{1 + \theta^S} Y_n^S \right) + (Y_n^i - \beta^M \gamma_n^i - \beta^S \gamma_n^i) + \sum_{j \in \{M, S\}} (X_{nj}^j - Y_n^j) + y_n^j

= y_n^i + y_n^M + y_n^S + \sum_{j \in \{M, S\}} (X_{nj}^j - Y_n^j).$$

We can therefore express GDP in country $n$ as:

$$y_n = C_n + I_n + R_n = X_n^i + \sum_{j \in \{M, S\}} (Y_n^j - X_n^j)

= C_n + I_n + N_n,$$

where, recall, $N_n$ is country $n$’s total net exports across all three sectors.

### 4. Quantitative experiments

As a first step in exploring the quantitative implications of this model, we have developed a numerical version of it using Matlab for a world of three hypothetical countries. We label the countries the United States, Germany, and China since our parameterization is meant to capture some key features of these economies. The numerical implementation allows us to examine the model’s general equilibrium implications for trade in manufactures, tangible services, and intangible services.
as well as for sectoral employment, aggregate income, and welfare. Of particular interest is understanding how these outcomes vary as we change deep parameters in the model.

Table 4.5 reports our baseline values for parameters that are common across countries (such as $\theta_j$ and $\rho_j$). Table 4.6 reports baseline values for parameters that are country specific (such as $L_i$ and $T_i^{E}$). Table 4.7 displays key outcomes generated by our baseline parameters. The first row of the table shows labor income. World income (normalized to 100) serves as our numéraire. Where possible we have chosen parameter values that are common in the literature (such as $\theta^M = 4$) or that can be calibrated directly to data (such as $\beta^{MS} = 0.16$ based on Input-Output Tables). The relative size of the three economies is largely determined by $L_i$, with China’s labor force twelve times Germany’s and four times that of the United States. We have chosen the iceberg costs ($d_{nl}^M = 1.7$ and $d_{nl}^S = 3.0$, for $n \neq l$) so that the model yields plausible outcomes for the fraction of manufacturing output (22 per cent) and services output (3 per cent) that is traded. In some cases the parameters are new to this model (such as $\rho^M = 0.8$, $h_{il}^M = 1.7$, $h_{il}^S = 2.0$ for $l \neq i$), and we get little direct guidance from data without a more formal calibration strategy, which is beyond the scope of this chapter.

The technology parameters ($T_i^{E}$ and $T_i^{D}$) play a central role in our analysis, and the model helps us understand the mapping between them and basic observable outcomes. These parameters vary along three dimensions, which we consider in turn. Along the country dimension, we have chosen the technology parameters to deliver the large real income advantage of the United States over China, while keeping Germany slightly below the level of the United States. We report these outcomes in the second to fourth rows of Table 4.7. Real income adds royalty income to wage income, while real consumption, in turn, subtracts investments in intangibles. Along the industry dimension, we have chosen the technology parameters to deliver Germany and China’s specialization in manufacturing relative to the United States. These outcomes appear in the sectoral employment shares in Table 4.7. The final dimension is between exclusive and diffused...
technology. Here we gave the United States an advantage in diffused technology to capture the large magnitude of U.S. intangible services exports. The outcomes for international trade in Table 4.7 show the United States running a large trade deficit in manufacturing and a large surplus in intangible services.

Having described our baseline, we now consider some numerical experiments. The first moves exclusive U.S. manufacturing technologies to U.S. diffused technologies. Specifically, we hold all other parameters at their baseline values while, for the United States, we lower $T_M^E$ from 10 to 0 while raising $T_M^D$ from 4 to 14.

China is the big beneficiary of this shift, as China’s low wage allows it to exploit manufacturing technology originating in the United States. China’s real wage increases nearly 20 per cent. The U.S. real wage doesn’t fall, and real income actually rises by 3 per cent as royalties rise. While income effects are positive, the US manufacturing sector shrinks dramatically. The manufacturing share of employment falls by 3 percentage points and U.S. exports of manufactures shrivel.

<table>
<thead>
<tr>
<th>Table 4.6 Country-specific parameters</th>
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<tbody>
<tr>
<td></td>
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<tr>
<td><strong>United States</strong></td>
</tr>
<tr>
<td>Labor endowment</td>
</tr>
<tr>
<td>Investment in intangibles</td>
</tr>
<tr>
<td>Aggregate trade deficit</td>
</tr>
<tr>
<td>Exclusive technology:</td>
</tr>
<tr>
<td>manufacturing</td>
</tr>
<tr>
<td>services</td>
</tr>
<tr>
<td>Diffused technology:</td>
</tr>
<tr>
<td>manufacturing</td>
</tr>
<tr>
<td>services</td>
</tr>
<tr>
<td>Iceberg costs:</td>
</tr>
<tr>
<td>manufacturing (importer):</td>
</tr>
<tr>
<td>United States</td>
</tr>
<tr>
<td>Germany</td>
</tr>
<tr>
<td>China</td>
</tr>
<tr>
<td>services (importer):</td>
</tr>
<tr>
<td>United States</td>
</tr>
<tr>
<td>Germany</td>
</tr>
<tr>
<td>China</td>
</tr>
<tr>
<td>Diffusion costs:</td>
</tr>
<tr>
<td>manufacturing (receiving):</td>
</tr>
<tr>
<td>United States</td>
</tr>
<tr>
<td>Germany</td>
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<tr>
<td>China</td>
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<tr>
<td>services (receiving):</td>
</tr>
<tr>
<td>United States</td>
</tr>
<tr>
<td>Germany</td>
</tr>
<tr>
<td>China</td>
</tr>
</tbody>
</table>
The correlation of diffused technology $\rho$ is a prominent (yet elusive) parameter in our model. To explore its role, we repeat the first experiment, but now with $\rho^M = 0$ (rather than $\rho^M = 0.8$). In this scenario, diffused technology will have very different realizations depending on where it is put to use in production. With $\rho^M = 0$ we find that U.S. income is even higher and the U.S. manufacturing sector remains more competitive in world markets. China experiences roughly the same increase in its real wage, and Germany’s also rises a bit. The lower value of $\rho^M$ raises the benefit of diffusion since ideas experience a greater transformation as they spread. A diffused idea is thus less substitutable across locations.

5. Conclusion

While we have applied our framework to a hypothetical three-country, three-sector world, the apparatus is flexible enough to deal with an arbitrary number of countries and alternative sectoral breakdowns. We’ve kept the analysis here static, but activity in the intangibles sector could be tied to the expectation of future royalty earnings as in EK (1999), EK (2001), and EK (2007).

These extensions are conceptually fairly straightforward. A barrier to future quantitative work in these directions is the remaining gap between key concepts in the theory and the data that are reported. The availability of data on services trade has improved dramatically in the last few years, but separating its tangible and intangible components remains a daunting challenge. Overcoming that

<table>
<thead>
<tr>
<th>Table 4.7 Baseline outcomes</th>
<th>United States</th>
<th>Germany</th>
<th>China</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate labor income</td>
<td>48.9</td>
<td>16.0</td>
<td>35.1</td>
</tr>
<tr>
<td>Real income per worker</td>
<td>7.6</td>
<td>6.6</td>
<td>1.6</td>
</tr>
<tr>
<td>Real consumption per worker</td>
<td>6.0</td>
<td>6.0</td>
<td>1.4</td>
</tr>
<tr>
<td>Real wage</td>
<td>5.1</td>
<td>4.3</td>
<td>1.2</td>
</tr>
<tr>
<td>Sectoral employment shares (%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>manufacturing</td>
<td>17.0</td>
<td>22.0</td>
<td>25.0</td>
</tr>
<tr>
<td>services</td>
<td>64.0</td>
<td>70.0</td>
<td>66.0</td>
</tr>
<tr>
<td>intangibles</td>
<td>19.0</td>
<td>7.7</td>
<td>8.8</td>
</tr>
</tbody>
</table>

International trade:

<table>
<thead>
<tr>
<th>Manufacturing:</th>
<th>United States</th>
<th>Germany</th>
<th>China</th>
</tr>
</thead>
<tbody>
<tr>
<td>imports</td>
<td>8.1</td>
<td>5.3</td>
<td>4.2</td>
</tr>
<tr>
<td>exports</td>
<td>4.2</td>
<td>5.0</td>
<td>8.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tangible services:</th>
<th>United States</th>
<th>Germany</th>
<th>China</th>
</tr>
</thead>
<tbody>
<tr>
<td>imports</td>
<td>1.0</td>
<td>0.9</td>
<td>0.0</td>
</tr>
<tr>
<td>exports</td>
<td>0.2</td>
<td>0.1</td>
<td>1.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Intangible services:</th>
<th>United States</th>
<th>Germany</th>
<th>China</th>
</tr>
</thead>
<tbody>
<tr>
<td>imports</td>
<td>0.0</td>
<td>0.1</td>
<td>5.9</td>
</tr>
<tr>
<td>exports</td>
<td>4.8</td>
<td>1.3</td>
<td>0.0</td>
</tr>
</tbody>
</table>
challenge is essential to understanding the role of creativity in driving growth in the world economy.

Notes
* We thank Yuta Watabe for excellent research assistance. We thank Costas Arkolakis, Lili Yang, and an anonymous reviewer for very helpful comments.
1 According to Reinsdorf and Slaughter (2009), whether a transaction under this fourth mode constitutes a service export depends on the length of stay of the natural person. If she stays over a year she becomes a resident of the destination so her service is considered part of the gross domestic product of the destination.
2 See Santacreu (2016).
3 Corrado et al. (2009) include in their definition of intangible investment both worker training and advertising, which are quantitatively important in the U.S. economy. The assets created by such investments, human capital in the first case and firm goodwill in the second, would seem rival. While we don’t model either training or advertising here, we would exclude such investments and subsequent assets from our concept of intangibles. We think of worker training, like education in general, as contributing to the supply of skills to be treated in the accounts accordingly. The goodwill a company creates through advertising, while an asset to the investing firm, is also a liability for its competitors. We treat the net contribution as zero. We have trouble envisioning an economy that could grow simply through the proliferation of ads, unless we take the view that an ad can make a consumer happier with a given physical allocation.
4 We extracted these figures from the OECD National Accounts. The sample are the 20 countries with data going back to 1985. Lipsey (2009) provides a depressing account of myriad conceptual and practical problems associated with the measurement of service trade. In particular he documents how services trade in earlier periods may have been more severely underreported, rendering the apparent rise in services trade illusional. Borga (2009) reports how the United States Bureau of Economic Analysis (BEA) has changed how it measures U.S. trade in three service sectors.
5 Reported figures may significantly understate U.S. service exports. See in particular Mutti and Grubert (2009), Robbins (2009), Moris (2009), McIver and Prescott (2010), and Guvenen et al. (2017). A reason is that U.S. tax policy makes it advantageous for U.S. corporations to shift profits overseas, creating an incentive to underreport exports of intangible services. The U.S. Bureau of Economic Analysis reports U.S. foreign investment receipts of a similar magnitude to U.S. service exports, suggesting a large upper bound on the degree of underreporting.
6 See Timmer et al. (2015) for a description of the data.
7 To quote van der Marel and Shepherd (2013a), “It is well known that services data become increasingly inaccurate as they are disaggregated.”
8 We are, of course, not the first to apply gravity analysis to trade in services. Without needing data on bilateral trade flows in services, Jensen and Kletzer (2005) assess the tradability of different service sectors by looking at the concentration of occupations employed extensively in these sectors across U.S. Metropolitan Statistical Areas (inferring that greater concentration is made possible by greater tradability). Lejour and Verheijden (2007) make use of bilateral data on services trade from the Canadian provinces and the European Union to estimate a gravity model for services. Egger et al. (2012) use data from the Organization for Economic Cooperation and Development to estimate a structural gravity model of trade in goods and services. Anderson et al. (2014) compare the role of distance and national borders in trade in goods and services among Canadian provinces and between individual provinces and the United States. Anderson et al. (2016) use OECD data to estimate a structural gravity equation for 12 individual service sectors. Van der Marel and
Shepherd (2013a, 2013b) examine how regulation affects the tradability of services using a dataset developed by Francois and Pindyuk (2013), combining data from Eurostat, the International Monetary Fund (IMF), the OECD, and United Nations. Gervais and Jensen (2013) estimate a model of services trade among U.S. states based on differences between demand and supply at different U.S. BEA labor market areas. Miroudot et al. (2016) examine the effect of services trade on measured productivity in services.

Pursuing the example above, we treat a current Netflix viewer of the Big Sleep as consuming the tangible services of Netflix and Netflix as purchasing the intangible services of Warner Brothers as an input into its streaming services.

One can conceive of much more complex patterns of diffusion. For simplicity we stick here with this simple dichotomy.

Consider the iPhone. While various rival inputs are used to produce it and deliver it to consumers (production labor, glass, aluminum, and employees at the Apple store), much of its value is due to the non-rival intangible assets embodied in it (engineering, software, and sleek design). The abstraction of our model is to lump these multiple dimensions of the non-rival inputs into a single intangible that we call the “technology.”

As discussed in Arkolakis et al. (2014), for \( \rho = 0 \), an idea provides an efficiency \( Q_l > q_l \) in only one location \( l \). Everywhere else the efficiency is \( q_l \).

References


CEPII (2017) Geography Database (website).


Trade in goods and trade in services


Appendix A

Deriving the efficiency distributions

Here we derive the distributions of efficiency posited in the text from more primitive assumptions about the discovery and diffusion of ideas, extending the model in EK (2001) to incorporate technology diffusion.

Creators in country $i$ generate ideas about how to produce some variety in some sector $j \in \{M, S\}$ at different locations $l$ in the world. By date $t$ the number of ideas originating in country $i$ for a variety in sector $j$ is distributed Poisson with parameter $\bar{a}T_{i,j}$. In what follows we consider a particular variety within a particular sector $j$ at a particular date $t$ and, for parsimony, drop the $j$ superscript and $t$ subscript.

Once diffused, an idea from origin $i$ enables the variety to be produced in different countries $l$ with efficiencies $Q_{li}$, for $l = 1, \ldots, N$. As in Arkolakis et al. (2014), we assume that these efficiencies are realizations from the joint distribution:

$$
Pr[Q_{li} \leq q_1, Q_{l2} \leq q_2, \ldots, Q_{lN} \leq q_N] = F_D(q_1, q_2, \ldots, q_N)
$$

where $\rho \in [0, 1)$ and $h_{li} \geq 1$ with $h_{ii} = 1$. To insure that this distribution is nonnegative we assume that it is defined only for

$$
q_l \geq q_i = q/h_{li}
$$

where:

$$
q = \bar{a}^{-1/\theta}N^{(1-\rho)/\theta}.
$$

Note that:

$$
F_D(q_1, q_2, \ldots, q_N) = 0.
$$

Sending $q_l \to \infty$, $l' \neq l$, the marginal distribution is:

$$
F_{li}^D(q_l) = 1 - \bar{a}^{-1}(q_lh_{li})^{-\theta}.
$$
Evaluating the marginal distribution at the lower bound:

\[ F^D_l(q_l) = 1 - N^{-(1-\rho)}, \]

implying a mass at the lower bound of the marginal distribution.\(^\text{12}\)

Upon its creation, an idea may not be immediately available for production. The technology diffuses over time to ever greater sets of countries. While the dynamics of diffusion could be quite general, for our purposes here we’ve limited diffusion to a one-step process. An idea is initially “exclusive” and available only in the country where it originated. Then, at some random date, it “diffuses,” becoming available to all countries. The date at which it diffuses is independent of the values of the \(Q_l\)'s the idea delivers. Hence the distribution of the \(Q_l\)'s is the same regardless of whether the idea has diffused or not.

Say that the probability that an idea from \(i\) has diffused is \(p_i\). Defining \(T^D_i = p_i T_i\) and \(T^E_i = (1 - p_i) T_i\), it follows that the number of ideas that have diffused is distributed Poisson with parameter \(\bar{a} T^D_i\) while the number that remain exclusive is distributed Poisson with parameter \(\bar{a} T^E_i\). The efficiencies of these two sets of ideas are independent of one another.

An idea from \(i\) that’s diffused has an efficiency distribution \(F^D_i(q_1, q_2, \ldots, q_N)\) given by (26). An idea from country \(i\) that has not diffused, since it is only available for use in country \(i\), has an efficiency distribution derived from (26) by letting \(q_l \to \infty\) for all \(l \neq i\); that is:

\[ F^E_i(q) = 1 - \bar{a}^{-1} q^{-\theta}, \]

for \(q \geq \bar{a}^{-1/\theta}\).

The number of exclusive ideas from \(i\) with efficiency above \(z\) is distributed Poisson with parameter:

\[ \bar{a} T^E_i \left[ 1 - F^E_i(z) \right] = T^E_i z^{-\theta}, \]

so that \(\bar{a}\) cancels out. By letting \(\bar{a} \to \infty\) we get full support \(z \geq 0\).

We can follow a similar strategy to derive the joint distribution of the best ideas that have diffused from country \(i\). With probability \(1 - F^D_i(z_1, z_2, \ldots, z_N)\) an idea from \(i\) has efficiency exceeding \(z_1, z_2, \ldots, z_N\) in at least one location \(l\). Hence the number of ideas from \(i\) that provide efficiencies above \(z_1, z_2, \ldots, z_N\) in at least one location is distributed Poisson with parameter:

\[ \bar{a} T^D_i \left[ 1 - F^D_i(z_1, z_2, \ldots, z_N) \right] = T^D_i \left( \sum_{l=1}^N (z_l h_l)^{-\theta/(1-\rho)} \right)^{1-\rho}. \]

Once again \(\bar{a}\) drops out. Again, by letting \(\bar{a} \to \infty\) this distribution applies to the positive orthant.
Appendix B

Deriving conditional probabilities

If country $n$ obtains a variety of tangible sector $j$ produced using technology that diffused from country $i$, what is the probability that country $l$ is the producer of this variety for country $n$? Since different countries $l$ may find the very same technique to be the lowest cost, our approach in the text, which relied on independent Poisson distributions, won’t work in this context. Instead, this derivation starts from the joint complementary distribution of costs across producing countries $l$ when using technology that’s diffused from $i$.

Denote the lowest cost, using a technology diffused from $i$ to serve $n$ from location $l$, by $C_{nli}^D$. From (7), their joint complementary distribution across production locations $l$ is:

$$G_{nli}^D(c_1, c_2, \ldots, c_N) = \Pr[C_{n1l}^D > c_1, C_{n2l}^D > c_2, \ldots, C_{nNl}^D > c_N]$$

$$= \Pr[Z_{n1l}^D \leq \frac{b_i^l d_{nl}^l h_{nl}^l}{c_1}, Z_{n2l}^D \leq \frac{b_i^l d_{nl}^l h_{nl}^l}{c_2}, \ldots, Z_{nNl}^D \leq \frac{b_i^l d_{nl}^l h_{nl}^l}{c_N}]$$

$$= \exp \left( -T_i^D \left( \sum_{j=1}^{N} \left( b_i^l d_{nt}^l h_{nt}^l \right)^{-\theta l / (1 - \rho l)} c_{nt}^{\theta l / (1 - \rho l)} \right)^{1 - \rho l} \right).$$

(29)

Differentiating (29) with respect to its $l$'th argument and evaluating at $c_l = c$ for $l = 1, \ldots, N$, we get:

$$\frac{\partial G_{nli}^D}{\partial c_l} = -\exp \left( -T_i^D \left( \sum_{j=1}^{N} \left( b_i^l d_{nt}^l h_{nt}^l \right)^{-\theta l / (1 - \rho l)} c_{nt}^{\theta l / (1 - \rho l)} \right)^{1 - \rho l} \right)$$

$$\times (1 - \rho l) T_i^D \left( \sum_{j=1}^{N} \left( b_i^l d_{nt}^l h_{nt}^l \right)^{-\theta l / (1 - \rho l)} c_{nt}^{\theta l / (1 - \rho l)} \right)^{-\rho l} c^{-\rho l / (1 - \rho l)}$$

$$\times \left( b_i^l d_{nt}^l h_{nt}^l \right)^{-\theta l / (1 - \rho l)} \frac{\theta l}{1 - \rho l} c^{\theta l / (1 - \rho l) - 1},$$
which simplifies to:

\[
\frac{\partial \tilde{G}_{n,l}^{D}(c, c, \ldots, c)}{\partial c_l} = -\exp \left( -T_l^{D} \left( \sum_{j=1}^{N} \left( b_j^l d_{al-j}^l h_j^l \right)^{-\rho_l/(1-\rho_l)} \right)^{1-\rho_l} c_l^{\rho_l} \right)
\]

\[
\times T_l^{D} \left( \sum_{j=1}^{N} \left( b_j^l d_{al-j}^l h_j^l \right)^{-\rho_l/(1-\rho_l)} \right)^{-\rho_l/(1-\rho_l)} \left( b_j^l d_{al-j}^l h_j^l \right)^{-\rho_l/(1-\rho_l)} c_l^{\rho_l-1}
\]

\[
= \frac{\left( b_j^l d_{al-j}^l h_j^l \right)^{-\rho_l/(1-\rho_l)} dG_{n,l}^{D}(c)}{\sum_{j=1}^{N} \left( b_j^l d_{al-j}^l h_j^l \right)^{-\rho_l/(1-\rho_l)}}
\]

where, we define:

\[
\bar{G}_{n,l}^{D}(c) = \exp \left( -T_l^{D} \left( \sum_{j=1}^{N} \left( b_j^l d_{al-j}^l h_j^l \right)^{-\rho_l/(1-\rho_l)} \right)^{1-\rho_l} c_l^{\rho_l} \right).
\]

Since \( \bar{G}_{n,l}^{D}(c) \) is itself a complementary distribution (the probability that all the \( C_{n,l}^{D} \) for \( l = 1, \ldots, N \), exceed \( c \)), we can see that \( \bar{G}_{n,l}^{D}(0) = 1 \) and \( \bar{G}_{n,l}^{D}(\infty) = 0 \).

Setting all but the \( l \)th argument in (29) to 0, we obtain the marginal complementary distribution of \( C_{n,l}^{D} \):

\[
G_{n,l}^{D}(c) = \exp \left( -T_l^{D} \left( b_j^l d_{al-j}^l h_j^l \right)^{-\rho_l} c_l^{\rho_l} \right).
\]

Hence:

\[
g_{n,l}^{D}(c) = -\frac{dG_{n,l}^{D}(c)}{dc}
\]

is the corresponding density.

The probability that \( l \) is the lowest cost supplier, conditional on delivering to \( n \) at unit cost \( c \), is:

\[
g_{n,l}^{D} = \frac{\partial \tilde{G}_{n,l}^{D}(c, c, \ldots, c)}{\partial c_l} / \bar{G}_{n,l}^{D}(c)
\]

Unconditionally, the probability that \( l \) is the lowest cost supplier is therefore:

\[
\pi_{n,l}^{D} = \int_{0}^{\infty} \pi_{n,l}^{D}(c) g_{n,l}^{D}(c) dc
\]

\[= - \int_{0}^{\infty} \frac{\partial \tilde{G}_{n,l}^{D}(c, c, \ldots, c)}{\partial c_l} dc
\]

\[= - \frac{\left( b_j^l d_{al-j}^l h_j^l \right)^{-\rho_l/(1-\rho_l)}}{\sum_{j=1}^{N} \left( b_j^l d_{al-j}^l h_j^l \right)^{-\rho_l/(1-\rho_l)}} \int_{0}^{\infty} \frac{dG_{n,l}^{D}(c)}{dc}
\]

\[= \frac{\left( b_j^l d_{al-j}^l h_j^l \right)^{-\rho_l/(1-\rho_l)}}{\sum_{j=1}^{N} \left( b_j^l d_{al-j}^l h_j^l \right)^{-\rho_l/(1-\rho_l)}}.
\]
Appendix C
Deriving the distribution of markups

Denote the unit cost of the lowest cost supplier of a variety to market $n$ as $C_n^{(1)}$ and the unit cost of the second-lowest cost (potential) supplier as $C_n^{(2)}$. If $\sigma^j < 1$ there is no finite monopoly price so that the markup is simply:

$$M_n^j = \frac{C_n^{(2)}}{C_n^{(1)}}.$$

If $\sigma^j > 1$ the monopoly price is a markup:

$$\bar{m}^j = \frac{\sigma^j}{\sigma^j - 1},$$

over $C_n^{(1)}$. The markup the seller will charge is thus:

$$\tilde{M}_n^j = \min\{M_n^j, \bar{m}^j\}.$$

To derive the distribution of this markup, it’s useful to condition on a cost $c_2$ such that $C_n^{(1)} < c_2 < C_n^{(2)}$. Defining:

$$\tilde{M}_n = \frac{c_2}{C_n^{(1)}},$$

the distribution of $\tilde{M}$ under this condition is:

$$\Pr[\tilde{M}_n \leq m] = \frac{\Phi^j(1 - m^{-\theta^j}) c_2^{\theta^j} \exp[-\Phi^j(1 - m^{-\theta^j}) c_2^{\theta^j}] \cdot \exp[-\Phi_n^j(c_2/m)^{\theta^j}]}{\exp[-\Phi_n^j c_2^{\theta^j}] \cdot \Phi_n^j c_2^{\theta^j}}.$$

Looking at the right-hand side, the first term in the numerator is the probability of exactly one cost in the interval between $m^{-1}c_2$ and $c_2$. The second term in the numerator is the probability that $C_n^{(1)} \geq c_2/m$. The term in the denominator is the probability that $C_n^{(1)} < c_2 < C_n^{(2)}$. Simplifying, this expression becomes:

$$\Pr[\tilde{M}_n \leq m] = 1 - m^{-\theta^j}.$$
Key for what follows is that this distribution does not depend on $c_2$. It follows that the distribution of $M^j_n$ is:

$$H(m) = \Pr[M^j_n \leq m] = 1 - m^{-\theta^j},$$

(31)

which is independent of $C^{(2j)}_n$. Taking account of the upper bound $\bar{m}^j$, we get the distribution of $\bar{M}^j_n$. 

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*Trade in goods and trade in services* 121
Appendix D

Deriving the price index

To derive the price index consider the price of a variety of tangible sector $j$, with second lowest cost $C^{(2)j}$ (we return to our convention of dropping the variety index, $\omega$). We can write its price $P_{jn}$ as:

$$P_{jn} = \begin{cases} C^{(2)j} & M_{jn}^j \leq \bar{m}^j \\ \bar{m}^j C^{(2)j}/M_{jn}^j & M_{jn}^j > \bar{m}^j \end{cases}$$

The independence of $M_{jn}^j$ and $C^{(2)j}$ allows us to write:

$$(p_j)_{1-\sigma} = \int_0^{\infty} c^{1-\sigma} dG^{(2)j}_n(c) \cdot \left[ \theta^j \int_1^{\infty} m^{-\sigma j-1} dm + \theta^j m^{\sigma j(1-\sigma)} \int_{m^j}^{\infty} m^{\sigma j-\theta j-2} dm \right]$$ (32)

where $G^{(2)j}_n$ is the distribution of $C^{(2)j}_n$. Since $C^{(2)j}_n$ lies below a cost $c$ only if there are two or more below $c$, this distribution is:

$$G^{(2)j}_n(c) = \Pr[C^{(2)j}_n \leq c] = 1 - \exp(-\Phi_n^j c^{\theta j}) - \Phi_n^j c^{\theta j} \exp(-\Phi_n^j c^{\theta j}),$$

that is, 1 minus the probability that there are zero or one. The corresponding density is:

$$dG^{(2)j}_n(c) = \theta^j \left( \Phi_n^j \right)^2 c^{2\theta j-1} \exp(-\Phi_n^j c^{\theta j}) dc.$$  

Attacking the first expression in (32):

$$\int_0^{\infty} c^{1-\sigma} dG^{(2)j}_n(c) = \theta^j \left( \Phi_n^j \right)^2 \int_0^{\infty} c^{2\theta j-\sigma j} \exp(-\Phi_n^j c^{\theta j}) dc$$

$$= \Gamma \left( 2\theta^j - (\sigma^j - 1) \right) \Phi_n^{-\sigma^j/\theta^j},$$ (33)
where $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$ is the gamma function. Attacking the second term in (32):

$$
\theta^j \int_1^{\bar{m}/j} m^{-\theta/j-1} dm + \theta^j \bar{m}^{(1-\sigma)}\int_{\bar{m}/j}^{\infty} m^{\sigma/j-\theta/j-2} dm = (1 - (\bar{m}/j)^{-\theta/j}) + \frac{\theta^j}{\theta^j - (\sigma^j - 1)} (\bar{m}/j)^{-\theta/j} = 1 + \frac{\sigma^j - 1}{\theta^j - (\sigma^j - 1)} (\bar{m}/j)^{-\theta/j}
$$

giving us the price index:

$$
p_n^j = \gamma^j (\Phi_n^j)^{1/\theta^j}, \quad (34)
$$

where:

$$
\gamma^j = \Gamma\left(\frac{2\theta^j - (\sigma^j - 1)}{\theta^j}\right)^{1/(1-\sigma^j)} \left(1 + \frac{\sigma^j - 1}{\theta^j - (\sigma^j - 1)} (\bar{m}/j)^{-\theta/j}\right)^{1/(1-\sigma^j)}.
$$
Appendix E

Deriving the royalty share

Having derived the price index for sector $j$ in destination $n$, we now turn to the royalties generated there. Consider a variety $\omega$ in sector $j$ in country $n$ with a price $P(\omega)$. Our CES demand system implies that its sales there are:

$$X(\omega) = A_n^j P(\omega)^{1-\sigma^j}$$

where:

$$A_n^j = \frac{X_n^j}{(p_n^j)^{1-\sigma^j}}.$$

Here $X_n^j$ is total absorption of sector $j$. The cost of tangible inputs to produce this variety are:

$$I(\omega) = \frac{X(\omega)}{M(\omega)}.$$

Input costs can be expressed as:

$$I(\omega) = A_n^j \left[ \frac{C(2)\omega}{M(\omega)} \right]^{1-\sigma^j} M(\omega) \leq \bar{m}^j$$

$$I(\omega) = A_n^j \left[ \frac{C(2)\omega}{M(\omega)} \right]^{1-\sigma^j} (\bar{m}^j)^{\sigma^j} M(\omega) > \bar{m}^j.$$

Integrating across varieties $\omega$, using (33), input costs in sector $j$ in country $n$ are:

$$I^j_n = A_n^j \int_0^\infty c^{1-\sigma^j} dG(2)\omega(c) \left[ \int_1^{m^j} m^{-1} dH(m) + (\bar{m}^j)^{-\sigma^j} \int_{m^j}^\infty m^{\sigma^j-1} dH(m) \right]$$

$$= A_n^j \Gamma \left( \frac{2\theta^j - (\sigma^j - 1)}{\theta^j} \right) \phi_n^{-(1-\sigma^j)\theta^j} \theta^j \left( 1 + \frac{\sigma^j - 1}{\theta^j - (\sigma^j - 1)} (\bar{m}^j)^{-\theta^j} \right).$$
Combined with the price index (17), this complicated expression reduces to:

\[
\frac{I^j_n}{X^j_n} = \frac{\theta^j}{1 + \theta^j}.
\]

It follows that intangible services (embodied in \(n\)’s absorption of tangible sector \(j\) goods) are:

\[
X^{ij}_n = X^j_n - I^j_n = X^j_n - \frac{\theta^j}{1 + \theta^j} X^j_o
\]

\[
= \frac{1}{1 + \theta^j} X^j_n.
\]

(35)